

Investigating the maths inside:

Prawns for profit

Information for teachers



*Maths Inside* is a project funded by the Commonwealth Department of Education and Training under the Australian Maths and Science partnership Programme.

The aim of *Maths Inside* is to increase engagement of students in mathematics by using rich tasks that show the ways mathematics is used in real world applications.

# About this module

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This module consists of the video ‘Prawns for profit’ and the following activities:

Activity 1 The perfect prawn? Years 9, 10

Activity 2 Farming prawns Years 7­–10

Activity 3 Populations, people and prawns Years 7–10

Activity 4 Selective breeding Years 7–10

Activity 5 Pop-up prawn pavilion Years 7–10, 11 General

# Feedback

Feedback from teachers about these classroom activities would be appreciated. Please complete the form at <http://tiny.cc/mathsinsidefeedback>.

Activity 1: The perfect prawn?

Students compare data on the mass of a sample of 40 prawns, taken from two different prawn farms. They decide which farm should be recommended as the new supplier of prawns for their seafood restaurant, supporting their decision with appropriate statistics.

Farm 1 harvests one crop of prawns per year and Farm 2 harvests two crops per year. There is then an opportunity for a discussion about why different farms would choose to grow one or two crops of prawns.

Data from a third farm then needs to be considered once all of the work has been done to make a choice between Farm 1 and Farm 2.

# Background

The CSIRO has been involved in the development and growth of prawn farms in Queensland as a viable industry. They have provided significant support in the area of selective breeding to improve the size and quality of the prawns grown.

The video should be watched first to provide the introduction and background that students need to investigate the idea of the perfect prawn.

This link provides additional information on how the Gold Coast prawn farm began working with CSIRO. <http://www.csiro.au/en/Research/AF/Areas/Aquaculture/Premium-breeds/Black-tiger-prawn>)

This link goes to the website of one of the prawn farms, north of Townsville, which harvests two crops of prawns per year <http://www.crystalbayprawns.com.au/>

There is further information on prawns and a prawn size chart at <http://australianprawns.com.au/types-of-australian-prawns/>

# Why do this?

This activity provides an opportunity for students to use their knowledge of statistics, mean, median, quartiles and boxplots to compare the production from different prawn farms.

Students are asked to use the results of their comparison to make a written recommendation about the farm that should win the contract as the new supplier of prawns to their seafood restaurant.

The activity is best done by students using a graphics calculator to compare the two sets of data provided. It can also be done using a spreadsheet but will require much more teaching of, or research into, the use of spreadsheets to draw box plots and curves for normal distributions.

An extension can be done to investigate normal curves.

# Australian Curriculum links

***Year 10 Statistics and Probability***

Determine quartiles and [interquartile range](http://v7-5.australiancurriculum.edu.au/glossary/popup?a=M&t=Interquartile+range) [(ACMSP248)](http://v7-5.australiancurriculum.edu.au/curriculum/contentdescription/ACMSP248)

Construct and interpret box plots and use them to compare [data](http://v7-5.australiancurriculum.edu.au/glossary/popup?a=M&t=Data) sets [(ACMSP249)](http://v7-5.australiancurriculum.edu.au/curriculum/contentdescription/ACMSP249)

Compare shapes of box plots to corresponding histograms and dot plots [(ACMSP250)](http://v7-5.australiancurriculum.edu.au/curriculum/contentdescription/ACMSP250)

***Year 10A Statistics and Probability***

Calculate and interpret the [mean](http://v7-5.australiancurriculum.edu.au/glossary/popup?a=M&t=Mean) and [standard deviation](http://v7-5.australiancurriculum.edu.au/glossary/popup?a=M&t=Standard+deviation) of [data](http://v7-5.australiancurriculum.edu.au/glossary/popup?a=M&t=Data) and use these to compare [data](http://v7-5.australiancurriculum.edu.au/glossary/popup?a=M&t=Data) sets [(ACMSP278)](http://v7-5.australiancurriculum.edu.au/curriculum/contentdescription/ACMSP278)

# Getting started

In the video, the perfect prawn is described as being one that grows fast, survives well, tastes nice and breeds well. It also fits into the 80th percentile for all of these characteristics. So the prawn is perfect from a farming perspective rather than a consumer’s perspective.

## Distributions

The introductory activity, arranging the students by height using two methods, is designed to get students thinking about describing data using the statistics that they know. It is meant to be a visual activity and to encourage class discussion.

## How big?

This activity is not totally necessary but should lead to a discussion of ratio. Students should discover that there are approximately twice as many prawns per kilogram compared to the size category. The sizing chart was developed when pounds, not kilograms, were used to weigh prawns. It may help students develop a picture of the size of prawns and what size might be suitable for their restaurant.

# Two different prawn farms

Information about two different prawn farms is provided in either a word document or a spreadsheet (MI\_Prawns\_Activity1\_PerfectPrawn\_TwoFarmsData).

Farm 1 is a typical farm from the Gold Coast of Queensland or northern New South Wales which produces one harvest of prawns per year.

Farm 2 is typical of a farm north of Townsville (such as Crystal Bay Prawns) which produces two harvests of prawns. The consistently warmer climate supports this and gives farms the advantage of being able to provide fresh prawns over a longer time period.

For different groups of students to use different sets of data, the spreadsheet titled *Mass generator* (MI\_Prawns\_Activity1\_PerfectPrawn\_MassGenerator) recalculates the data automatically. It may be easier to do this first and provide different groups of students with different sets of data. The prawn number is provided to make it easier for students to enter the data accurately into a graphics calculator.

It may be necessary to teach students how to use their graphics calculators for statistics.

The data for Farm 1 will give an average of about 35 grams.

The data for Farm 2 will give an average of about 25 grams but as they harvest each crop at one time, the prawns are generally between 20 and 30 grams.

Once they start investigating the data, students should take into account the average mass, the minimum, the first quartile, the median, the third quartile, the maximum, the range and the interquartile range and draw suitable graphs. Box plots would be most suitable. If using a spreadsheet, it may be necessary to search the internet for instructions or use a YouTube video such as <https://www.youtube.com/watch?v=ucWmfmXb1kk>

# What about this company?

The Farm 3 data has a much greater spread than the other two farms. Students should be able to see this from an inspection of the data, and find there are relatively even numbers of larger and smaller prawns. This data is found either in a word document or a spreadsheet (MI\_Prawns\_Activity1\_PerfectPrawn\_Farm3Data).

# Answers

It is expected that most students will choose Farm 1 as their preferred provider. Even though the cost of the prawns is higher, the prawns are larger and more even in size, as required for the restaurant. Students should be able to justify this with their calculations and comparisons of the different boxplots.

They may also suggest to have Farm 1 as the main supplier but buy from Farm 2 or Farm 3 occasionally and freeze the prawns so that they have the smaller prawns on hand when required. If they choose Farm 3 because of the price, they should justify a degree of wastage because of the differences in the size of the prawns. Students must be able to justify their recommendation based on the data and graphs that they have generated.

Answers for the given data are provided below.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Farm 1** | **Farm 2** | **Farm 3** |
| *Mean* | 34.8 | 24.1 | 29.1 |
| *Minimum* | 33.1 | 20.1 | 20.1 |
| *1st quartile* | 33.8 | 21.7 | 23.15 |
| *Median* | 34.8 | 23.3 | 30.05 |
| *3rd quartile* | 36.0 | 26.3 | 34 |
| *Maximum* | 36.9 | 29.9 | 39.5 |
| *Standard deviation* | 1.18 | 2.81 | 5.93 |

The further discussion on why different prawn farms choose to harvest one or two crops of prawns each year could include:

* The length of the growing season because of the difference in the climate between the Gold Coast and northern Queensland
* The cost of producing and marketing the prawns for different target markets as found from their research above
* Profitability related to doing everything twice compared to once and the potential price of the prawns grown.

Again, an internet search may provide other discussion points.

# Extension Answers

The 80th percentile will be the 32nd prawn in the list when the list is arranged in order.

|  |  |  |
| --- | --- | --- |
| Farm 1 | Farm 2 | Farm 3 |
| 36.0 | 27.0 | 34.5 |

In reality, you would expect a normal distribution. Even though the prawns are harvested in different ways, it is expected the sizes will be evenly distributed around the mean, in the same way as the heights of students and many other randomly occurring variables.

# Resources

Graphics calculator

Data for Farm1 and Farm 2 (MI\_Prawns\_Activity1\_PerfectPrawn\_TwoFarmsData)

Data for Farm 3 (MI\_Prawns\_Activity1\_PerfectPrawn\_Farm3Data)

Mass Generator spreadsheet (MI\_Prawns\_Activity1\_PerfectPrawn\_MassGenerator).

You could also run the mass generator a number of times and provide students with printouts for the two prawn farms.

## Additional resources

A search of the internet will provide a number of videos on YouTube on how to use a graphics calculator for statistics and plotting box plots.

<https://www.youtube.com/watch?v=ucWmfmXb1kk> provides instruction on how to create a boxplot in Excel.

<https://www.youtube.com/watch?v=UASCe-3Y1to> is a good place to start with drawing a frequency distribution histogram in Excel. Each set of data will require different ‘bin limits’ which can be time consuming for drawing normal curves. To draw a normal distribution curve, select ‘insert an X-Y scatter plot as a line’ instead of the bar graph. Care is needed here as the limited amount of data may mean it is not a nice smooth normal curve.

<http://apfa.com.au/prawn-farming/> There is a significant amount of information about prawn farming on this website.

Activity 2: Farming prawns

# Background

Australia’s population is predicted to reach 46 million by 2075. Among the many challenges this will bring is the need for Australia to continue to secure its own food supply, contribute to the food supply of the region, and be competitive in global food markets.

The CSIRO partnered with Australian prawn farmers to breed a prawn that is now producing record farm yields. CSIRO breeding, through domestication of prawns, has added roughly a 20% increase to the supplies of top quality, sustainably produced seafood.

This activity looks at some of the challenges involved in prawn farming and the mathematics required to support the management of a prawn farm. The following gives a reasonably brief outline of prawn farming.

<https://www.business.qld.gov.au/industries/farms-fishing-forestry/fisheries/aquaculture/species/black-tiger-prawn/growing-harvesting>

# Why do this?

Prawn farming is a relatively new industry to Australia and the management and stocking of ponds to farm prawns is an application of a real situation that will allow students to use their knowledge and skills related to measurement, ratio, the arithmetic mean and interpreting graphs. *How big is a hectare?* could be done as a stand-alone activity to review area calculations; *Stocking the ponds* and *Baby prawns* provide opportunities to check arithmetic calculations; and *Feeding* is an opportunity to test or develop students’ understanding of ratio.

# Australian Curriculum links

***Year 7 Statistics and Probability***

Calculate [mean](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=mean), [median](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=median), [mode](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=mode) and [range](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=range) for sets of [data](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=data). Interpret these statistics in the context of [data](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=data) [(ACMSP171)](http://www.australiancurriculum.edu.au/curriculum/contentdescription/ACMSP171)

Describe and interpret [data](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=data) displays using [median](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=median), [mean](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=mean) and [range](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=range) [(ACMSP172)](http://www.australiancurriculum.edu.au/curriculum/contentdescription/ACMSP172)

***Year 8 Number and Algebra***

Solve a [range](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=range) of problems involving rates and ratios, with and without digital technologies [(ACMNA188)](http://www.australiancurriculum.edu.au/curriculum/contentdescription/ACMNA188)

***Year 8 Measurement and Geometry***

Choose appropriate units of measurement for [area](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=area) and [volume](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=volume) and convert from one unit to another [(ACMMG195)](http://www.australiancurriculum.edu.au/curriculum/contentdescription/ACMMG195)

Find perimeters and areas of parallelograms, trapeziums, rhombuses and kites [(ACMMG196](http://www.australiancurriculum.edu.au/curriculum/contentdescription/ACMMG196))

***Year 9 Measurement and Geometry***

Calculate areas of composite shapes [(ACMMG216)](http://www.australiancurriculum.edu.au/curriculum/contentdescription/ACMMG216)

Calculate the surface [area](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=area) and [volume](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=volume) of cylinders and solve related problems [(ACMMG217)](http://www.australiancurriculum.edu.au/curriculum/contentdescription/ACMMG217)

Solve problems involving the surface [area](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=area) and [volume](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=volume) of right prisms [(ACMMG218)](http://www.australiancurriculum.edu.au/curriculum/contentdescription/ACMMG218)

***Year 10 Statistics and Probability***

Investigate and describe bivariate numerical data where the independent variable is time [(ACMSP252)](http://www.australiancurriculum.edu.au/curriculum/contentdescription/ACMSP252)

# How big is a hectare?

The average pond area used for farming prawns is approximately 1 hectare (ha) and the average depth is 1.2 to 1.5 m, giving a volume of approximately 12 to 15 megalitres (ML). This activity is designed to familiarise students with the hectare as a unit of area. The old unit of acres is also included, as this is a unit still used in some situations.

Teachers may need to demonstrate how to use DaftLogic ([www.daftlogic.com](http://www.daftlogic.com)) which allows areas to be calculated using Google Earth. Once on the DaftLogic site students need to find the Google Maps Area Calculator Tool by scrolling down the page. They then enter the address in the ‘search’ bar. There is no need to sign up for an account unless you wish to save the areas.

To investigate the size of housing blocks currently being sold, students could calculate the size of their housing block or a familiar block of land.

# Stocking the pond

Students need to understand what is meant by the mean, and how the mean can be used to represent the data and to help make decisions.

The number of prawns can also be calculated using the capture-recapture method.

(See *Activity 3: Counting bees* in the *Bees with backpacks* module for more details of the capture-recapture technique.)

# Baby prawns

Some of the video is filmed alongside the tanks that hold about a million prawn larvae. Students may need to be reminded that 1 m3 has a capacity of 1000 litres (1kL).

# Feeding

The feeding of prawns is not calculated on a per prawn basis but by way of a food conversion ratio (FCR). Ideally, to maximise return, this ratio should be approximately 1.5:1, that is 1.5 kg of food for every 1 kg of prawns.

Using a spreadsheet, students determine the amount of feed required for ‘their’ pond each week, and then find the total amount of feed used in their pond during a 24-week period. Twenty-four weeks is the approximate time for the prawn larvae to grow to full size, ready for harvesting and sale.

The FCR can then be calculated by dividing the total amount of food by the total weight of the prawns. This value is independent of the pond used.

# Answers

**How big is a hectare?**

1 ha = 10 000 m2 (100 m x 100 m)

1 ha = 2.47 acres

1 acre = 4047 m2 acre = 1012 m2

The average size of a block of land varies between states and locations within states. However, blocks as small as 200 m2 are not uncommon.

The three largest ponds are Pond 3 with an area of 1.53 ha, then Pond 1 with 1.42 ha, and finally Pond 8 with 1.29 ha.

|  |  |  |  |
| --- | --- | --- | --- |
| **Pond number** | **Approximate area** | **Pond number** | **Approximate area** |
| 1 | 1.42 | 6 | 0.87 |
| 2 | 1.05 | 7 | 0.97 |
| 3 | 1.53 | 8 | 1.29 |
| 4 | 0.95 | 9 | 0.89 |
| 5 | 0.42 | 10 | 0.94 |

Note that these measures are approximate; answers will vary depending on how accurately students work.

**Stocking the ponds**

Example using Pond 1:

1.42 ha = 1.42 x 10 000 = 14 200 m2

Number of prawns = 14 200 x 40 = 568 000

Mean number of prawns, sampled from the edge = 52.3

Number of prawns per m2 = 52.3 ÷ 1.44 = 36.3

Total number of prawns in the pond = 36.3 x 14 200 = 515 460

Mean number of prawns, sampled from the middle = 53.1

Number of prawns per m2 = 53.1 ÷ 1.44 = 36.9

Total number of prawns in the pond = 36.9 x 14 200 = 523 980

The difference between the two means is small at 0.8, and represents a difference of about 8000 prawns. Students could discuss the significance of this difference in terms of the total number of prawns in the pond, but given that casting at the edge is easier, this probably does give a sufficiently accurate estimate of the number of prawns in the pond. If considered in terms of cost, 8000 prawns weigh approximately 280 kilograms and at $20 a kilogram this could be regarded as significant.

**Baby prawns**

Volume of tank = (4.3 x 1.8 x 1.2) + (π x 0.92 x 1.2)

= 9.288 + 3.954

= 12.342 m3

Capacity of tank = 12.342 x 1000 = 12 342 L

Number of prawns per litre = 1 000 000 ÷ 12 342 = 81

Approximate number of larvae in ten-litre bucket = 81 x 10 = 810

Mean number of larvae in ten-litre bucket = 818.6

Number of larvae in tank = 818.6 ÷ 10 x 12 342 = 1 010 300

Number of prawns required for Pond 1 = 568 000

Number of ten-litre buckets required = 568 000 ÷ 818.6 = 694

**Feeding**

Total weight of feed = Number of prawns Mass of prawn FCR

= 568 000 x 35 x 1.5 g

= 568 000 x 35 x 1.5 ÷ 1000 kg

= 13 250 kg

The spreadsheet (MI\_Prawns\_Activity2\_Farming\_FeedingProgramAnswers) can be used to generate answers for other ponds. Note that the FCR will be the same for every pond.

Students will need to rearrange the *Total weight of feed* formula to calculate the FDR in their spreadsheet. They can either be given the rearranged formula, or encouraged to work it out themselves.

Note that the FDR is above the optimum of 1.5. Reducing the quantity of feed would reduce the FDR but could lead to the prawns growing more slowly or dying.

The prawns grow slowly at first but grow more quickly after 7–8 weeks. The growth is consistent each week until the growth slows after 22 weeks.

The fact that the growth slows after 22 weeks when the prawns are about 35 g indicates that it would be a good time to harvest, as the need for feed continues to grow.

The main assumption made is that there has been no recognition of the survival rate of the prawns. The above assumes that the all prawns survive for the 24 weeks. The survival rate will be dependent on the quality of the prawns, feeding, the weather, water quality and the avoidance of disease.

# Resources needed

Students will need access to the internet (research and DaftLogic) plus a spreadsheet.

# Further ideas

The publication *Australian Prawn Farming Manual: Health Management for Profit* is the product of a diverse range of people with extensive experience in both the Australian and international shrimp farming industry.

It is available at: <http://aciar.gov.au/files/node/737/Australian%20prawnfarming%20manual%20final.pdf>

Activity 3: Populations, people and prawns

# Background

Australia’s population is increasing, along with our demand for cooked prawns. Approximately 50% of prawns consumed in Australia are imported from countries such as China and Vietnam. Meanwhile the populations of prawns and other marine species in the wild are decreasing.

CSIRO scientists have bred ‘the perfect prawn’ and developed a new plant-based prawn feed.

After watching the video, students will:

* investigate historical data and the current population of Australia
* calculate percentage changes in populations
* use a calculator and spreadsheets to model exponential growth and decay
* make predictions of future populations
* examine why it is necessary to seek alternative sources of prawns

# Why do this?

Australians love catching and eating seafood!

According to the Australian Bureau of Statistics (ABS) it is estimated that over five million Australians take part in recreational fishing in Australia as a leisure activity. Therefore many high school students participate.

Many marine species in and around Australia have declining populations. In some cases this is primarily due to commercial and recreational harvesting.

The activities provide students the opportunity to use percentages, graphs and spreadsheets to represent real data that changes over time and make predictions about the future. Teachers can also use this context as a vehicle for introducing the magical number ‘e’ to students.

# Australian Curriculum links

# *Year 7 Number and Algebra*

# Find percentages of quantities and express one quantity as a [percentage](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=Percentage) of another, with and without digital technologies.[(ACMNA158)](http://www.australiancurriculum.edu.au/curriculum/contentdescription/ACMNA158)

Investigate, interpret and analyse graphs from authentic [data](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=Data) [(ACMNA180)](http://www.australiancurriculum.edu.au/curriculum/contentdescription/ACMNA180)

***Year 8 Number and Algebra***

Investigate the concept of irrational numbers [(ACMNA186)](http://www.australiancurriculum.edu.au/curriculum/contentdescription/ACMNA186)

Solve problems involving the use of percentages, including [percentage](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=Percentage) increases and decreases, with and without digital technologies [(ACMNA187)](http://www.australiancurriculum.edu.au/curriculum/contentdescription/ACMNA187)

Solve a range of problems involving rates and ratios, with and without digital technologies [(ACMNA188)](http://www.australiancurriculum.edu.au/curriculum/contentdescription/ACMNA188)

***Year 10 Statistics and Probability***

Investigate and describe [bivariate numerical data](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=Bivariate+numerical+data) where the [independent variable](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=Independent+variable) is time [(ACMSP252)](http://www.australiancurriculum.edu.au/curriculum/contentdescription/ACMSP252)

Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative [data](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=Data) [(ACMSP253)](http://www.australiancurriculum.edu.au/curriculum/contentdescription/ACMSP253)

***Year 10A Statistics and Probability***

Use information technologies to investigate [bivariate numerical data](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=Bivariate+numerical+data)sets. Where appropriate use a straight line to describe the relationship allowing for variation [(ACMSP279)](http://www.australiancurriculum.edu.au/curriculum/contentdescription/ACMSP279)

# Getting started

Initially it might be interesting to ask students the following stimulus questions without allowing them access to the internet. It might also be interesting to ask students to write their answers on a piece of paper and then award prizes to the best estimators.

* What do you think the current population of Australia might be?
* How long do you think it might take for the current population of Australia to double?
* What do you think the population of Australia might be in the year of your 80th birthday?

With regard to the following questions it would be good to ask students who engage in recreational fishing (and those who know adults who work in commercial fishing) about the rules and regulations regarding certain marine species in waters close to your school.

* Why are prawns, lobsters and oysters expensive?
* With regard to recreational fishing, what is meant by the phrase ‘bag and size limits’?
* Why are strict laws and fines in place for people who catch and keep too many lobsters, for example, or lobsters which are below the minimum legal size?

# Modelling populations

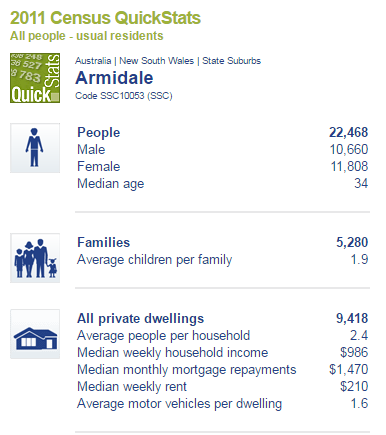
## The population of Australia

Point out how important the census is for keeping track of Australia’s population.

This webpage allows people to look at census statistics for a suburb, town or a postcode.

[www.abs.gov.au/websitedbs/censushome.nsf/home/quickstats?opendocument&navpos=220](http://www.abs.gov.au/websitedbs/censushome.nsf/home/quickstats?opendocument&navpos=220)

For example:



Most of the class discussion will follow on from the answers. There is the opportunity to use spreadsheets, which are provided.

# Answers

## The population of Australia: Milestones

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Year** | **1918** | **1959** | **1981** | **2004** | **2007** | **2010** | **2013** | **2016** |
| ***Time*** | 0 | 41 | 63 | 86 | 89 | 92 | 95 | 98 |
| ***Population***  ***(millions)*** | 5 | 10 | 15 | 20 | 21 | 22 | 23 | 24 |

## Australia’s population clock

Answers will vary as the clock changes frequently.

Note that the questions are asking for ‘every hour’ so students will need to convert the figures from ‘1 person every 5 minutes’, say, to ’12 people per hour’.

## The growth rate of populations

The calculator procedure for repeatedly increasing or decreasing by a percentage works on most calculators, including the calculators in mobile phones.

### Growth of a town

* 5100 people after one year
* 5520 people after five years
* It takes just over 35 years for the population to double
* There are 6095 people after ten years
* Same answer of 6095 using the simpler method.

At first the number of people increases slowly, but as time progresses the increase each year is substantially more than the previous year.

Some reasons forthe percentages varying so much in these early years include:

* The population was smaller. In the 1789 to 1790 every 100 arrivals increased the population by 15.5%
* People died in large numbers (due to disease and malnutrition) and new arrivals (convicts and settlers) came in large groups on boats.

Note: Indigenous Australians were not included in the population until 1966.

### Growth of Australia (early)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Year** | **1788** | **1789** | **1790** | **1791** | **1792** | **1793** | **1794** | **1795** |
| ***Population*** | 859 | 645 | 2056 | 2873 | 3264 | 3514 | 3579 | 3466 |
| ***Population growth*** | na | -214 | 1411 | 817 | 391 | 250 | 65 | -113 |
| ***Population growth (% to 1 d.p.)*** | na | -24.9 | 218.8 | 39.7 | 13.6 | 7.7 | 1.8 | -3.2 |

### Growth of Australia (recent)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Year** | **2006** | **2007** | **2008** | **2009** | **2010** |
| ***Population*** | 20 627 547 | 21 016 121 | 21 475 625 | 21 865 623 | 22 172 469 |
| ***Population growth*** | na | 388 574 | 459 504 | 389 998 | 306 846 |
| ***Population growth (% to 1 d.p.)*** | na | 1.9 | 2.2 | 1.8 | 1.4 |

### Growth of Australia (from 1900)

In the graph below, note the ‘dip’ in 1940 and 1950 due to World War II and then the post war baby boom and migration.

Answers will vary from student to student.

### Growth of Australia (future)

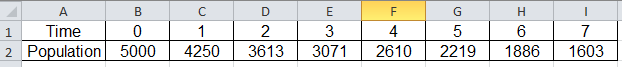
The spreadsheet on percentage growth (MI\_Prawns\_Activity3\_Populations\_PercentageGrowth) can be used to match the existing percentage growth curve to predict future populations. The mean annual percentage growth rate from 1972 to 2011 was 1.35%. The lowest was 0.89% and the highest was 2.08%.

Other assumptions used by the ABS include fertility, mortality and overseas migration.

## The ‘wild’ population of fish, prawns and other species

Changes in population are usually expressed as a percentage.

If population decreases by 15% per annum, by the end of each year it will be 85% of the previous year. The percentage 85% can be expressed as the decimal 0.85.



# Extension: The number called *e*

Show the video (MI\_Prawns\_Activity3\_Populations\_Video). It explains how to use an Excel spreadsheet to draw an exponential curve which can be used as a model Australia’s population data. It also shows the equation of the curve.

The equation includes the number *e*, known as Euler’s number, in honour of the mathematician Leonard Euler (pronounced ‘oiler’) (1707-1783). He is considered one of the greatest mathematicians of all time.

Provide students with the spreadsheet which approximates *e* (MI\_Prawns\_Activity3\_Populations\_Approx)

In the video, the equation used as a model for the population of Australia was

y = 3,826,798.533e0.016x

Scientists use the number *e* in exponential models, rather than 2 or 3 or some other number, because functions involving *e* are quite simple to differentiate. The derivative of the equation could be used to calculate an estimate of the population growth at any time in the past, or to make a prediction about the population growth at some time in the future.

Some interesting facts about Euler’s number: [www.mathsisfun.com/numbers/e-eulers-number.html](http://www.mathsisfun.com/numbers/e-eulers-number.html)

# Resources needed

* Calculators (mobile phone calculator will suffice)
* Internet access
* Access to devices with basic spreadsheet capabilities (laptop or desktop computer, tablet, graphics or CAS calculator)
* Instructions for setting up the spreadsheet (MI\_Prawns\_Activity3\_Populations\_SSInstructions)
* Spreadsheet for finding percentage growth (MI\_Prawns\_Activity3\_Populations\_PercentageGrowth)
* Video about drawing exponential curves (MI\_Prawns\_Activity3\_Populations\_Video)
* Spreadsheet for approximating *e* (MI\_Prawns\_Activity3\_Populations\_Approx)

Activity 4: Selective Breeding

# Background

This activity could possibly be delivered to students as a cross-curricular STEM activity involving biology and/or agriculture teachers.

The students will be calculating the inbreeding coefficient. A worked example is provided in the student activity sheet.

# Why do this?

Inbreeding can have significant positive effects in a population. Prawn farmers breed for positive effects such as size, growth rate, taste, resistance to disease etc.

Inbreeding can also have adverse effects. Farmers and animal breeders need to consider the implications of their breeding over a number of generations especially when breeding for particular traits or trying to breed out particular defects. There also may be economic impacts as a result of their breeding decisions.

This is a practical application of probability and indices.

# Australian Curriculum links

***Year 8 Number and Algebra***

Use index notation with numbers to establish the index laws with positive integral indices and the zero index (ACMNA183)

***Year 10 Statistics and Probability***

Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence (ACMSP246)

This task also satisfies many of the criteria of the proficiency strands.

**Understanding**

Students build a robust knowledge of adaptable and transferable mathematical concepts. They make connections between related concepts and progressively apply the familiar to develop new ideas. Students build understanding when they describe their thinking mathematically and when they interpret mathematical information.

**Problem-solving**

Students develop the ability to interpret and investigate problem situations, and communicate solutions effectively. Students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations and when they verify that their answers are reasonable.

**Reasoning**

Students develop an increasingly sophisticated capacity for logical thought and actions, such as proving, evaluating, explaining and justifying. Students are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached and when they prove that something is true.

# Getting started

It is highly recommended that maths teachers consult with biology and/or agriculture teachers prior to delivering this lesson. It is quite likely that students will steer the conversation in the direction of inbreeding in the human population, so some degree of preparation for that is advisable.

As a beginning activity, the genetic simulation to observe the effects of inbreeding *What could go wrong?* shows the possible results of breeding including how dominant and recessive genes lead to defects. In the simulation, the prawns with defects are those that are not commercially viable.

# Answers

Due to the complexity of the task, answers as percentages are given with the question. Detailed diagrams are provided on the *Coefficient of inbreeding answers* sheet. Students are required to convince themselves and the teacher that they have reached the correct answer with good reason.

# Resources needed

Internet access

*What Could Go Wrong?*  instructions (this has a short list of equipment needed)

*Coefficient of inbreeding questions* sheet

*Coefficient of inbreeding answers* sheet

# Further ideas

Students may be interested in investigating what happens when inbreeding is common in the human population:

The problems associated with inbreeding are well documented in humans, especially in members of various royal families. Queen Victoria of the United Kingdom married her first cousin. The condition of hemophilia was passed to several of her ancestors. They then had children with members of other royal families in which there were also carriers of the condition. One of Victoria’s grandchildren died from blood loss at the age of two.

[www.sciencemag.org/news/2009/10/case-closed-famous-royals-suffered-hemophilia](http://www.sciencemag.org/news/2009/10/case-closed-famous-royals-suffered-hemophilia)

Charles II of Spain (1661 to 1700) was born to parents who were uncle and niece. This was done to stop ‘outsiders’ entering their royal family. He was physically and mentally disabled. His jaw was so badly deformed that he could barely speak or chew. This was possibly due to the inbreeding of his parents and previous generations of his family.

<https://en.wikipedia.org/wiki/Charles_II_of_Spain>

Inbreeding in other royal families:

<https://en.wikipedia.org/wiki/Royal_intermarriage>Puppy factories: Why are some activists opposed to the concept of puppy farms?

Thoroughbred horses. The magic millions.

Activity 5: Pop-up prawn pavilion

This activity is a feasibility study for a small business selling prawn skewers at a local market. Students investigate fixed costs, variable costs, break-even points and profit/loss, using spreadsheets and associated graphs. They make various changes to improve profitability, justifying their decisions.

# Background

A small business is an independently owned and operated company that is limited in size and revenue, with those limits depending upon the industry. The Australian Taxation Office defines a small business as one that has an annual turnover of less than $2 million dollars. Fair Work Australia defines a small business as having fewer that 15 employees.

A class discussion should conclude that a market stall falls well within the bounds of a small business. In regards to taxation it cannot be regarded as a ‘hobby’ because it is being set up with the intention of making a profit.

# Why do this?

This is an ideal opportunity for students to use a spreadsheet to investigate a financial scenario. The spreadsheet will enable them to rapidly change figures and see the outcome.

It is a practical application of linear equations and can be used to introduce or consolidate the equation of a straight line, y = mx + c. The fixed costs are represented by the y-intercept. The gradients will vary depending on the decisions that the students make.

Simultaneous equations can be addressed when investigating the break-even point.

# Australian Curriculum links

***Year 8 Number and Algebra***

Solve problems involving profit and loss, with and without digital technologies (ACMNA189)

***Year 10 Number and Algebra***

Solve problems involving linear equations, including those derived from formulas (ACMNA235)

Solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology [(ACMNA237)](http://v7-5.australiancurriculum.edu.au/curriculum/contentdescription/ACMNA237)

# Getting started

Have a class discussion about running a business or conducting a market stall to make a profit. The intention is to find out what knowledge the students already have about running a business and their experiences of markets. Discuss any parameters that you want placed on the running of the stall such as an agreement on the fixed costs and opening times on each weekend.

# Financial viability study

This can be done through class discussions and internet searches. It is important for students to understand the financial terms involved, such as start-up, break-even, profit, loss, fixed costs and variable costs.

Depending on the initial class discussions, the list of fixed costs could be agreed upon and the same for all. Students could then be allowed to decide their own variable costs based on their decisions about the recipe, the size of prawns used, and the cost of the ingredients.

The use of a spreadsheet allows the students to run a variety of scenarios to help them make decisions to ensure the profitability of their stall. This is an excellent opportunity to use the graphing capabilities of a spreadsheet for the students to develop a practical application of linear graphs.

# Borrowing money to get started

Students need to produce a convincing argument that shows the amount they intend to borrow. Their business plan should include the location of the stall, the fixed and variable costs, the break-even point, and for how long they need to run it to save the initial target of $5 000. Their argument should be backed up with appropriate graphs.

It may be necessary to adjust the initial target depending upon information gathered.

# Resources needed

Students will require access to the internet and a spreadsheet.