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What is a reasonable answer?

Ways for students to investigate and develop their number sense

TRACEY MUIR examines a framework for developing number sense and explains how it may be expanded to include measurement benchmarks. She explains how a class of Year 5/6 students grappled with some activities designed to promote number sense and finishes by providing an estimation game that may be applied to any classroom setting.

How high is Mount Everest? What is the mass of a blue whale? What is the population of China? What would be reasonable answers to these questions? Although number sense is difficult to define, it involves having a good intuition about numbers and their relationships, including the ability to have a 'feel' for the relative size of numbers and to make reasonable estimations. McIntosh, Reys and Reys (1992, p. 3) define it as "an ability to use numbers and quantitative methods as a means of communicating, processing, and interpreting information". Students with good number sense typically recognise the relative magnitude of numbers, appreciate the effect of operations on numbers and have developed a system of personal benchmarks (NCTM, 2000). In this article the concept of number sense will be further discussed, a framework for examining basic number sense will be revisited and activities and experiences to promote number sense will be shared, including examples from a lesson conducted with a Grade 5/6 class (aged 11–12 years).

What is number sense?

Number sense refers to a person's general understanding of number and operations, along with the ability and inclination to use this understanding in flexible ways (McIntosh, Reys & Reys, 1997). It involves having a sense of what numbers mean; people with a good sense of number have the ability to look at the world in terms of quantity and numbers, and have a sense, for example, of when 100 is a lot and when it is not much (Shumway, 2011). This knowledge can then be used to make comparisons, interpret data, estimate and answer the question, "Does that answer make sense?" (Shumway, 2011). Number sense is thought, therefore, to be highly personalised (McIntosh, et al., 1997) with its development being a lifelong process (Reys, Lindquist, Lambdin & Smith, 2007).

A framework for developing number sense

Twenty years ago, McIntosh et al. (1992) provided a framework for clarifying and organising the various components of basic number sense. It is perhaps timely to revisit this framework in terms of its relevance and applicability to today's teaching of mathematics and the imminent introduction of the Australian Curriculum. An overview of the framework is presented in Table 1 (for the complete version, please see McIntosh et al., 1992, p. 4).

In the original framework, the key components and understandings are numbered and a third column contains examples of mathematical concepts (e.g., in relation to the second key component and the understanding of 'mathematical properties', the concepts of commutativity, associativity, distributivity, identities and inverses are listed). In addition, the key components are further explained and examples given of how these understandings may be demonstrated. For example, benchmarks are referred to in the first key component, and these are further explained as "often used to judge the size of an answer, such as recognising that the sum of two 2-digit numbers is less than 200, that 0.98 is close to 1 or that $\frac{4}{9}$ is slightly less than $\frac{1}{2}$" (McIntosh, et al., 1992, p. 6). Benchmarks are also related to personal attributes or encounters, such as a person weighing 50 kg may use this information in estimating the weight of another person, or a regular attendee at an AFL football game may use crowd attendance as an estimate for judging the size of other crowds. The connection with measurement units, however, is not entirely explicit within the framework and the author of this article would recommend that in addition to 'Developing a sense of the relative and absolute magnitude of numbers'

Table 1. Overview of framework for considering number sense (adapted from McIntosh, et al., 1992).

Key component	Understandings
Knowledge of and facility with NUMBERS	• Sense of orderliness of numbers • Multiple representations for numbers • Sense of relative and absolute magnitude of numbers • System of benchmarks • Relationship between the size of the number and the size of the unit [added by author]
Knowledge of facility with OPERATIONS	• Understanding the effect of operations • Understanding mathematical properties • Understanding the relationship between operations
Applying knowledge of and facility with numbers and operations to COMPUTATIONAL SETTINGS	• Understanding the relationship between problem context and the necessary computation • Awareness that multiple strategies exist • Inclination to utilise an efficient representation and/or method • Inclination to review data and result for sensibility

and a ‘system of benchmarks’ (from the first key component), another understanding is included, which could be termed ‘Developing a sense of the relationship between the number of units and the size of the unit’. This would account then for the ability to determine what 1000, for example, means in terms of metres, kilometres, kilograms, days and so on.

Investigating number sense further

In order to gain an understanding of how students might demonstrate number sense, particularly in terms of their ‘knowledge and facility with numbers’, the following investigations were undertaken with a Grade 5/6 class. The investigations were particularly designed to provide students with the opportunity to demonstrate whether or not they had a ‘sense of the relative and absolute magnitude of numbers’ and a ‘system of benchmarks’ with which they could operate to make reasonable estimates. In addition to providing the teacher with information as to students’ use of these concepts, the activities also provided insights into other mathematical understandings including averages, place value, and measurement attributes, such as area, mass and length.

Three questions and three answers

The lesson began with the whole class sitting in front of the whiteboard. The lesson was introduced by posing three questions, one at a time, to the class and soliciting three different responses to each question. The questions were:

- How far is it from Launceston to Hobart?
- How far is it from the Earth to the sun?
- How old am I?

Three responses for each question were recorded on the board and students were then asked to vote for the most reasonable answer. Discussion then occurred on the reasons for the vote, what referents were used to make estimates, which questions were ‘easier’ to estimate and why. The activity was particularly useful for gauging which referents were used by students. For example, students indicated that they found it easier to estimate my age because they could gauge whether or not I was older than their parents, whereas distances posed more of a problem. By varying the questions, teachers can scaffold students’ attempts at estimation and encourage them to make connections with more familiar contexts (e.g., How long does it take you to travel to school? How many kilometres would this be?).

How big, how tall, how many?

The next activity required the students to provide (and receive) answers to a number of questions which did not have an obvious answer and involved larger numbers (e.g., How tall is Mount Everest? What is the population of China? How tall is the world’s tallest man? How long would it take you to count to a million?). The Guinness Book of Records (Glenday, 2007), Google and the Australian Bureau of Statistics (ABS) website (www.abs.gov.au) are all useful resources for questions. There needs to be enough questions for everyone in the class to have one each. The questions were put on cards and placed on students’ backs, without revealing the question (see Figure 1). Each student then asked three people to provide an answer to the question, and in

Figure 1. Estimation activity, How big, how tall, how many?

turn, provided answers to others' questions. The responses were recorded on a proforma (see Appendix A). After everyone recorded three responses, the class regrouped and answers were discussed. Before the students were allowed to look at their questions, they had to firstly decide on an appropriate answer, based on the responses received, and secondly, identify an appropriate question which would 'fit' the answer. The students were very curious as to what their questions were, and the 'correct' answers. Answers could either be provided by the teacher or as an additional challenge, students could find out the answers.

As to be expected, the responses the students gave and received varied, with the following being illustrative of the types of responses received.

In response to the question, "What is the record for the most friends on Facebook?" Jackson received the following three responses (reprinted as recorded on proforma):

- 1 million and 2
- 20 million
- 1 million

He recorded that "1 million and 55" would be a reasonable estimate, because "two people said about a million and the 20 million was just way out."[1] The selection of the 'middle answer' was a common strategy with others recording: "2 million because it was the middle answer" and "7 because it is in the middle of the answers that I got [had answers of 10, 6 and 8]," and "150 because all my answers were between 100–300 m". Others referred to the most common answer, "1, because that was the most popular". Such answers revealed an informal use of averages and this could be noted as an area for future teaching directions.

While most students provided a response to the question which asked them to designate a reasonable answer to the question, only some students attempted to identify an appropriate question to fit their answers. Students' answers also showed that they were more likely to nominate a question if the answers received were accompanied by a unit. For example, Susan received three responses to her question: 1000 m, 1000 m, and 500000 m. She chose "2000 m" as a reasonable answer because "it's what first popped into my head", and nominated that her question might be "How far is it from ____ to ____?" Although her answer does not indicate the use of 'average' as a strategy as other student examples showed, her nominated question does show that she related the measurements to length. Having said that, it is more likely that the answers were in reference to the length or height of something, rather than distance, as metres, rather than kilometres, were used (the question was actually 'How high is Mount Everest?' with the answer being 8848 m).

As expected, there were a huge range of answers received to the questions which also varied widely in their reasonableness. Discussion could also therefore occur around how students decided upon what answers to give, leading into an appreciation of the need to develop personal benchmarks in order to make more reasonable estimates.

What is a good estimate?

The last activity required students to answer an estimation challenge recorded on a card. The original source for this idea is unknown, but was adapted from a resource shared by a State Curriculum Officer at a workshop. The cards contain estimation challenges that are only limited by the teacher's imagination. Each card is laminated and has the challenge printed on the front, with the actual item placed on the back (see Figure 2). The following are examples of suitable challenges to use:

- This is a sheet from a roll of home brand toilet tissue which contains 280 sheets. Estimate in metres the total length of the roll.

1 The answer (as of May 2011) is 32 258 201 with the record being held by Eminem

- Tags like these are used for sealing bread packets. Estimate how many tags could be made from an A4 piece of plastic (see Figure 2).
- This is a freezer bag tie. Estimate how many ties you could make from a 1 metre piece of wire.

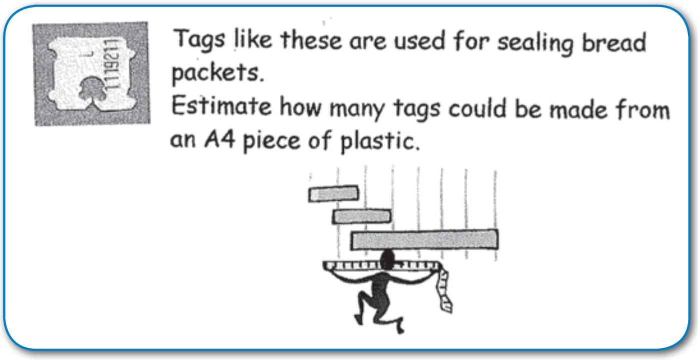

Figure 2. Estimation card.

Students worked in small groups to solve the challenge. They were asked to provide an estimated answer, explain how they decided upon their answer and to indicate whether or not they thought others would make the same estimate. In response to the freezer bag tie question, one group provided an estimate of "23" and explained, "the wire looked about five cm and five goes into 100 twenty times, then we added some extra". They were also able to say that others might make the same estimate, but that "it depends on how long they think it is".

As with the previous activity, the students' approaches and answers not only revealed information about their number sense, but insights into their other mathematical understandings as well. For example, some of the challenges required them to have an understanding of concepts such as area, perimeter, conversion of units and number operations (particularly multiplication).

Indicators of number sense

Classroom observations revealed that the students did not give reasonable estimates to many of the questions, indicating a general lack of a sense of the relative and absolute magnitude of numbers. In addition, when students were asked to explain how they decided upon their estimate, many just "guessed" and few made reference to any type of personal benchmark. This was perhaps not surprising in that many of the questions were deliberately chosen to be beyond the realm of many of their personal experiences. The activities did have the effect however, of making these understandings apparent and provided valuable information to the teacher on students' current number sense, particularly in relation to the magnitude of numbers and the relationship between the size of the number and the size of the unit.

The next section contains some examples of activities that could be used in the classroom to help develop students' number sense and the relevant understandings that were identified in Table 1.

Teaching for number sense

Students who find mathematics difficult often lack number sense, whereas students with strong number sense are often more adept with attempting problems and making sense of mathematics (Shumway, 2011). Students therefore need multiple opportunities to engage in number sense ideas, use number sense and discuss number sense ideas and strategies. In order to do this, authors such as Shumway (2011) and McIntosh et al. (1997) recommend the use of daily routines. Examples of such routines include:

Count around the circle

With students sitting in a circle, the teacher nominates a starting point to begin counting, a number to count by (e.g., start at 188 and count backwards by 10) and a student to begin. Before the counting commences, select a point approximately half way around the circle and ask the students to estimate or predict which number that person would say; reinforce the appropriate use of terms such as “somewhere,” “close” and “about.” Extend the questioning by asking why numbers such as 178 or 268 would not be good estimates. This activity can be varied and used on a daily basis (adapted from Shumway, 2011) and particularly supports a sense of the orderliness of numbers.

Subitising activities

Subitising, or the instant recognition of how many items are in a group, supports both the development of number sense and arithmetic ability (Clements, 1999). Encouraging students to instantly recognise small numbers of dot arrangements placed on cards or dominoes can be extended to estimating larger collections of items arranged in arrays, groups or randomly (see Figures 3 and 4). Ask questions such as: How many do you see? Did you need to count them all? Would there be more than…? Would there be less than…? etc. For larger collections, ‘vote’ on more than or less than amounts (e.g., hands up if you think more than 100). These activities would be particularly appropriate for developing a sense of the relative and absolute magnitude of numbers and for encouraging benchmarking.

Today’s number is…

This popular routine, which is used to develop students’ confidence with numbers and operations and to encourage them to describe patterns and relationships (McIntosh, De Nardi & Swan, 2000), can easily be adapted to focus on developing number sense. In addition to finding combinations of numbers which make 10, for example, students could be asked, “When is 10 big? When is 10 small? How far is it from 100?” (Shumway, 2011). The choice of numbers can be varied to include decimal fractions and common fractions with the aim being to develop benchmarks for these numbers (e.g., 0.99 —what is that close to? What would take approximately 0.99 of a second?). Developing a sense of place value could also occur through using the number ‘1’, for example, and looking at its value in numbers such as 10, 41 320, 0.01, 10, 1000, 0.12, etc. The activity particularly supports the understandings of multiple representations for numbers, magnitude and orderliness.

Counting the days in school

Depending upon the age of the students, Shumway (2011) identifies a number of ways in which this can be done and how it helps to develop a sense of number. For younger children, sticky notes placed on sentence strips (10 different coloured notes per strip) helps to develop counting sequences, the idea of numbers increasing, along with laying the foundation of an understanding of the base 10 system. Another way of keeping count is to place Unifix cubes or blocks in a container (1 cube = 1 day; 10 cubes can be joined together for counting in groups), while using an array model with 1–100 charts is particularly suitable for older students. This activity fosters a sense of (for example), “What does 100 look like?” and would help to facilitate the understanding of the relative magnitude of numbers.

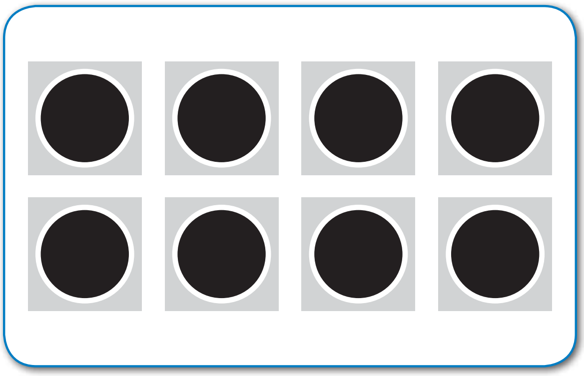

Figure 3. Dot card.

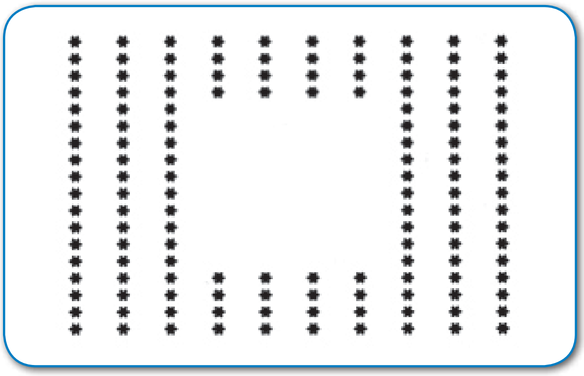

Figure 4. Array of stars.

Within limits

This activity as described by McIntosh et al. (2000) involves the teacher selecting something for the class to estimate (e.g., length of classroom, number of beans in a jar, the value of a pile of money), with small groups then discussing and agreeing on an upper and lower limit within which they are certain that the measurement lies. The limits are gradually reduced until everyone agrees on the range. For example, the lower and upper limits for "How many marbles in the jar?" may have started with more than one and less than 1000, but narrowed to more than 230 and less than 450. The activity is particularly useful for setting estimates within specific boundaries and for encouraging the use of 'average' (McIntosh et al., 2000).

Changing estimates (Shellshear, 2011)

Many estimation activities require students to make an estimate, then measure. This strategy encourages students to make an estimate, then change their estimate based on new information. For example, the question may be, "How many straws will it take to measure the length of the classroom?" Students make an initial estimate, then the teacher or another student begins measuring the length with straws (approximately 5). Students are then encouraged to change their estimate. This process continues, with estimates being revised 4–5 times until the answer is very close. The changing estimates can be recorded in a table (see Figure 5).

Using children's literature

There are a number of books which readily provide situations that encourage the development of number sense. Counting on Frank (Clements, 1990) is an obvious one, with other suitable stories including If the World were a Village (Smith, 2002), How Much is a Million? (Schwartz, 2004), Millions of Cats (Gag, 1996) and Math-terpieces (Tang, 2003). Resource books such as Exploring Mathematics Through Literature (Thiessen, 2004) include many examples of mathematically-related literature which could be read with a focus on developing number sense.

Concluding thoughts

Activities and experiences such as the ones outlined in this article have the potential to engage students in making sense of numbers, along with providing valuable information to the teacher about many other aspects of students' mathematical knowledge. The development of number sense is an essential part of 'being numerate' and relevant to the Australian Curriculum: Mathematics (ACARA, 2010) which mentions it explicitly in reference to number and algebra—"Students apply number sense and strategies for counting and representing numbers" (ACARA, 2010, p. 2)—and also in relation to describing appropriate learning opportunities for students in Years 3–6:

> …it is important for students to develop deep understanding of whole numbers to build reasoning in fractions and decimals and develop their conceptual understanding of place value. With these understandings, students are able to develop proportional reasoning and flexibility with number through mental computation skills. These understandings extend students' number sense and statistical fluency (ACARA, 2010, p. 5).

Challenge: How many straws will it take to measure the length of the classroom?				
Estimate 1	Estimate 2	Estimate 3	Estimate 4	Answer

Figure 5. Changing estimates table.

Furthermore, it would seem evident that engaging students in sense-making activities would support the development of both the content and proficiency strands, particularly as they include mention of "magnitude and properties of numbers" (ACARA, 2010, p. 2), and "use estimation to check reasonableness of answers" (e.g., pp. 27, 38).

It is hoped that this article has reminded readers how essential it is to develop number sense in our students, along with providing teachers with some practical suggestions on how this can be done. The number sense framework (McIntosh, et al., 1992) provides a useful overview for considering aspects of number sense and, along with the other resources identified, should provide a useful starting point for teachers who are looking to include regular number sense routines in their classrooms.

N.B. The answers to the questions at the beginning of the article are: Mount Everest is 8848 metres high, the mass of a blue whale is approximately 136 000 kg and the population of China in 2009 is 1.3 billion.

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Appendix: Estimation game

Your task:

To identify the question on your back and the answer to it!

Help others by giving them estimates for the questions on their backs.

Ask 3 people to give you an answer to your question and write their answers below:

1.

2.

3.

Based on their answers, what do you think would be the real answer (or a reasonable answer) to your question?

Why?

What do you think your question might be?

Why?

Your question is:

The answer is:

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