Discussion Paper

School mathematics for the 21st century

What should school mathematics of the 21st Century be like?

This paper outlines a broad framework for school mathematics in the 21st century. It is not related to a specific curriculum – in particular it does not refer to the current development of the National Mathematics Curriculum in Australia.

On National Mathematics Day 2009 (22nd May) the Australian Association of Mathematics Teachers (AAMT) invited teachers and others to consider this Discussion Paper. Within the Paper are some questions and discussion 'starters' that individuals and groups of teachers may wish to consider. There is no requirement to consider those questions, or any expectation that you will provide any feedback. If you do wish to make any comments, however, please send these to feedback@aamt.edu.au.

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Preamble — Mathematics for all

The *Standards for Excellence in Teaching Mathematics in Australian Schools* (2002, 2006)[1] are the profession's statement of what characterises the work of excellent teaching of mathematics. By providing the benchmark for 'excellence', the *Standards* provide a set of aspirational goals for all teachers of mathematics. One of the core components of excellent teachers' attributes — and therefore something that the AAMT believes should be part of the make-up of *all* teachers — is that they "believe all students can learn mathematics". This contrasts with much of what we see in the name of school mathematics — the system and structure is too often designed as a filter to separate those students who 'can' from those who 'can't', with only the former gaining real access to the valued and powerful mathematics.

This is not to say that all students should learn the same mathematical content. Eventually students' interests, enjoyment and preferred vocational trajectories will result in them taking different pathways in their study of mathematics. However in the 21st century, within those different mathematical pathways all students should experience rich and challenging mathematics as described later in this paper.

Believing that *all* students can learn mathematics has great significance if genuinely taken on as an underpinning principle in school mathematics, whether by individual teachers (the audience for the *Standards*), schools or whole education systems. Acknowledging that all people require mathematical skills for effective participation in their lives at home and in their communities, in education and training, and at work means that systems, schools and teachers need to take this principle seriously — sustained and well-informed effort from all parties can ensure that no students are left behind in their learning of mathematics.

Discussion starters:

1. *What are the implications of 'mathematics for all' for:*
    - *curriculum*
    - *teachers*
    - *schools*
    - *education authorities*
    - *the community*
2. *Education policy and curriculum documents have, for many years, promoted this notion of 'mathematics for all'. Why hasn't it been achieved? Is it really achievable?*

Why this paper

Mathematics is a way of looking at the world that provides people with tools to solve problems. As well as its contributions to human and social development, mathematics has aesthetic and cultural dimensions that make its study worthwhile in its own right. The AAMT believes that mathematics is a necessary inclusion in any curriculum for schools in the 21st century.

[1] Australian Association of Mathematics Teachers Inc. 2002. *Standards for Excellence in Teaching Mathematics in Australian Schools*. Adelaide: Author.

In *On the Shoulders of Giants*, eminent mathematician Lyn Arthur Steen captured the key contemporary challenge for mathematics curricula:

> To develop effective new mathematics curricula, one must attempt to foresee the mathematical needs of tomorrow's students. It is the present and future practice of mathematics — at work, in science, in research — that should shape education in mathematics. To prepare effective mathematics curricula for the future, we must look to patterns in the mathematics of today to project, as best we can, just what is and is not truly fundamental.[2]

The Executive and Council of the Australian Association of Mathematics Teachers believe it is important for those with an interest in school mathematics to discuss and reach some consensus on the nature and emphases necessary in school mathematics that is suitable for the 21st century.

A number of factors have influenced the development of this paper, and need to be central to any consideration of mathematics curriculum at this time and in the foreseeable future. These factors include:

- Mathematics — both the discipline itself, and its uses in other fields — has radically changed in the past 30 years, most notably through the development and uptake of powerful technologies for doing mathematics; school mathematics needs to reflect consideration of these changes.
- Extensive advances in the field of pedagogy, in particular to accommodate research into the brain and how it works; learning theories such as constructivism; ideas like multiple intelligences etc. make it crucial for the approaches to the teaching and learning of mathematics in schools to reflect these developments and advances.
- Contemporary views are that ours is a 'knowledge society'; education in general, and school mathematics in particular, need to respond to changes in the ways of being and working in the knowledge era.
- The emergence of 'numeracy' as a term that has been extensively adopted by politicians, policy makers and educators over the past 10 years in particular; school mathematics requires clear articulation in this context to ensure there is a common sense of purpose for school mathematics and numeracy as distinct yet connected educational constructs.
- The characteristics of young people born in the 1990s (so-called Generation Y); how they see themselves, what they expect from their education and how they learn all have a bearing on the nature of school mathematics and how students encounter it.

These factors are elaborated in the companion paper *Some key influences on school mathematics*[3].

Discussion starters:

1. *Are there other factors that need to be taken into account when thinking about school mathematics for the 21st century? If so, what might they be?*
2. *You might like to look at the video at http://www.youtube.com/watch?v=cL9Wu2kWwSY What implications do some of those figures have for school mathematics?*

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[2] Steen, L. A. 1992. *On the Shoulders of Giants: New Approaches to Numeracy.* National Academy Press: Washington DC.

[3] The sections of the companion paper are Changes in mathematics in the last 30 years; Learning mathematics as future citizens and workers in the information economy; Students' views of mathematics — deep and connected knowledge; Being numerate; and Generation Y.

The 'stuff' of Mathematics for the 21st century

Steen (1992) provides a frame for AAMT's thinking about aspects of mathematics that need to be the central to a curriculum that responds to current and broadly foreseeable contexts:

Recurring concepts (eg number, function, algorithm) call attention to what one must know in order to *understand* mathematics; common actions (eg represent, discover, prove) reveal skills that one must develop in order to *do* mathematics. Together, concepts and actions are the nouns and verbs of the language of mathematics.

Mathematical Concepts: These are the organisers of core 'content'. This core content could be elaborated as fully as might be desired. For this summary document a short description of each will be appropriate.

Mathematical Actions: The intention here is to identify the core 'actions' — the thinking, communicating and other activities — that constitute doing mathematics.

In this paper a third category is included to augment the *Mathematical Concepts* and *Mathematical Actions*. The so-called **Big Ideas in Mathematics** are key to connecting other aspects of mathematics, both between and within the *Mathematical Concepts* and *Mathematical Actions*. They are overarching ideas that are neither 'concepts' nor 'actions'. Most of the *Big Ideas* pervade a number of conceptual areas of mathematics, and provide connections between them. For example, notions of symmetry are evident in all four areas of *Mathematical Concepts*.

The way in which the *Big Ideas* can help learners to make connections can be illustrated by an example — *patterns* is one of the *Big Ideas*. Young children need to learn about place value as part of making sense of our number system; much of this learning hinges on them identifying and working with the *patterns* in the number system. When they are a little older, they extend the range of numbers they understand and deal with to include fractions — a focus on *patterns* is instrumental in their learning about fractions. Again and again as learners work to understand more and more complex mathematics (ie *concepts*) they can and should be working with the *patterns* that are central to that mathematics.

Some other ideas that are important in the teaching and learning of mathematics in the 21st century are difficult to fit into this framework of Big Ideas, Concepts and Actions. Included among these are 'mathematical literacies' (including 'statistical literacy') that see mathematical skills serving and supporting thinking, learning and work in other domains; and the need for students to appreciate to see mathematics as a very human endeavour that relies on historical and cultural perspectives, practices and conventions. These are taken to be over-arching themes for the framework.

Discussion starters:

1. *Are these really the 'big ideas' in mathematics? What would you describe as the 'big ideas of school mathematics'?*
2. *Are they (or some other list of 'big ideas') useful when thinking about school mathematics?*
3. *Does the list of 'big ideas' change for different levels of schooling? How do (some of) these 'big ideas' play out for students of different ages? (e.g. what does 'equivalence' look like for small children; in the middle years etc.)*
4. *The other category that is likely to be problematic is the list of 'mathematical actions'. Are there 'actions' you think are central to doing mathematics in schools that are omitted?*

Big ideas	Mathematical Concepts	Mathematical Actions
These ‘big ideas’ of mathematics are embedded across the mathematical concepts, providing the important connections between concepts.	These four mathematical concepts are the organisers of ‘content’ which may need to be elaborated to support teacher planning	These mathematical actions or processes are typically used to represent ‘doing’ mathematics. They are organised into four broad areas.
		Reasoning
	Quantity	Arguing, justifying and proving
		Generalising
Dimension		**Using procedures**
Symmetry		Comparing
		Calculating
	Shape and space	Measuring
Transformation		Operating (in the sense of ‘reflect in that line’ or ‘find the indefinite integral of…’)
Algorithm		**Communicating**
Patterns		Asking questions
		Visualising
	Uncertainty	Representing (numbers, visuals, symbols; and combinations)
Equivalence		Explaining
Representation		**Solving problems**
	Variables, relationships and change	Mathematical modelling
		Using mathematical tools