Submission to the Review of Stage 6 Mathematics Courses in New South Wales from the Executive of the Australian Association of Mathematics Teachers Inc.

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Executive summary

The key points in the AAMT submission are:

- Extensive and intensive consultation with teachers of mathematics, through the four teacher professional associations in NSW, should occur throughout the review and redevelopment of Stage 6 mathematics courses.
- In line with the needs of students as learners, workers and citizens in the 21st century Stage 6 courses need to promote and enable deep learning of mathematics.
- All students should have genuine access to a range of mathematics subjects that are challenging and rewarding, and that meet their needs and future aspirations.
- The amount of content in the revised Stage 6 courses should be limited to help ensure deep learning and the development of cohesive views of mathematics.
- A range of areas of mathematics that are currently not included in Stage 6 courses should be considered in terms of relevance and applicability in the 21st century.
- The classroom implementation of pedagogies that promote deep learning of mathematics should be encouraged in the documentation of the Stage 6 courses and, where feasible and appropriate, in the formal assessment of students.
- As part of the identification of content and processes for the revised Stage 6 courses consideration should be given to embracing hand-held mathematical devices with CAS capability.

Introduction

The Australian Association of Mathematics Teachers Inc. (AAMT) is the pre-eminent professional association in school mathematics and numeracy education in Australia. The Association exists to support and enhance the work of teachers, to promote the learning of mathematics and to promote progress in mathematics and numeracy education. The AAMT is a federation of state and territory organizations of teachers of mathematics. Four of the founding members of the AAMT are located in NSW — the Mathematical Association of NSW (MANSW), the Newcastle Mathematical Association (NMA), New England Mathematical Association (NEMA) and the Southern Cross Mathematical Association (SCMA). The AAMT believes that these organizations — and their members, the mathematics teachers of NSW — should be extensively consulted in the process of reviewing and renewing the Stage 6 syllabuses.

The time available for preparing this submission has not allowed detailed consultation with members and these affiliated associations. Further, the Executive believes that the input from the AAMT should be at a relatively general level and highlight important issues and trends that are evident in national and international developments in mathematics in the later years of schooling.

Mathematics for the 21st century

There is a paradox for mathematics in the 'knowledge era'. On the one hand it is clear that the pervasive technologies of the age are, and will continue to be, substantially based on, and enablers of, mathematics — mathematics is increasingly important in our society. Those developing technologies need, of course, to be highly mathematically competent. On the broader level as well our society is very much driven by data and analyses of mathematical models as a result of the technologies. Everyone needs mathematical skills and capabilities.

On the other hand, however, these technologies effectively 'submerge' the popular perception of what constitutes 'mathematics' — mathematics is, apparently, nowhere near as important as it used to be. No-one needs to be able to manually do a whole range of things such as simple and now, very complex, calculation as these can be automated

This paradox is resolved if we consider what we mean by mathematics. 'Low level' skills that can be more accurately and efficiently done by a machine are certainly much less relevant in use and can no longer be supported as the key outcomes of school mathematics *for their own sake*. The 21st century requires mathematics of a much higher order for citizens to be able to understand, work with and create mathematical models that are accessible and powerful in the context of current and emerging technologies. As a result, the important mathematics in schooling should be about this sort of mathematics.

That is not to say that formerly important 'lower level' skills are not important. In some, perhaps many cases they are, insofar as they are integral to being able to work with powerful mathematical tools, technologies and techniques. For example, the emergence of mental computation as an important component in young children's facility with number has clear practical uses. But mental computation is also important through its capacity to deepen understanding of how numbers 'work' and therefore critical to fluent use of mathematical models of all sorts.

There needs to be a major rethinking of school mathematics in the light of all this. Its purposes have to be in line with the mathematical needs of young people. Hence mathematics content needs to be selected for clear reasons that link to the nature of mathematics citizens need in and from schooling and into the future. These reasons do not include 'we have always done it.' Apparently heretical questions like "Should the kind of 'procedural' calculus that has been the pinnacle of achievement in school mathematics in the 20th century remain so in the 21st?" and "Does the funnelling of algebraic skills towards the goal of school calculus serve students' and the society's needs?" need to be considered.

Methodologies for teaching, learning and assessment also need to be critically analysed to ensure that the ways of working and learning are consistent with building in young people the kind of view of mathematics — and themselves as mathematicians — that will support their success in the knowledge era, whatever their vocational pathway(s).

Another key consideration in reviewing the Stage 6 courses in NSW goes beyond the broad areas of 'content' and 'process'. It is the dimension of how students think of mathematics and come to 'know' mathematics. Gilbert and Macleod (2006)[1] discuss the concept of 'deep approaches to learning' that are characterised by learning as 'making sense of physical concepts and procedures – coherent and integrated understanding sought'; as 'seeing phenomena in the world in a new way – developing ability to apply knowledge and methods to see phenomena in new ways, mediated by a reflective dimension'; and as 'change as a person – awareness of being able to see the world differently makes them see themselves differently'. Higher-order metacognitive skills also need to be given explicit attention and developed alongside learning content and process in order for students to successfully adopt these 'deep approaches to learning'.

In relation to learning mathematics at university in particular, these authors cite Crawford, Gordon, Nicholas and Prosser (1998, p. 465)[2] who concluded that '…fragmented conceptions of mathematics are associated with surface approaches to learning mathematics... On the other hand, cohesive conceptions of mathematics are associated with deep approaches to learning mathematics…. students holding cohesive conceptions of mathematics adopt deep approaches to learning mathematics, and have very different interpretations of learning mathematics.' This is likely to be true, at an appropriate level of sophistication, for learners of mathematics in our schools as well.

Hence a core characteristic of school mathematics needs to be for students to have the capacity to build cohesive conceptions of mathematics and be 'deep' learners of mathematics.

Students and mathematics

The above are general comments about the nature of mathematics and purposes for learning mathematics in school for *all* students. It is a strongly held principle of the AAMT that all students should have access to purposeful, challenging and personally rewarding mathematics courses of study.

Given that our young people have widely differing interests and future aspirations it is inevitable — indeed desirable — that there is a range of courses with different emphases and intentions to match these. The details of this range needs to be the subject of consultation in NSW.

Typically we see three broad types of course around Australia. These have been largely linked to likely post-school uses of mathematics and can be characterised as courses designed for students' likely post-school use of mathematics:

- Minimal later use of mathematics;

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[1] Gilbert, R. & Macleod, H. 2006. *An analysis of the current suite of QSA Years 11 and 12 syllabuses: A report for the Queensland Studies Authority Review of Syllabuses for the Senior Phase of Learning*. Brisbane: Queensland Studies Authority.

[2] Crawford, K., Gordon, S., Nicholas, J. and Prosser, M. (1998) Qualitatively different experiences of learning mathematics at university, *Learning and Instruction,* 8, 5, 455-468.

- Some possible use of mathematics in further learning and work; and
- Further learning and work has a strong mathematical component.

These broad categories are linked to previously held views of mathematics and its role in society. They may also be appropriate in some form or other into the 21st century, but this should not be assumed — serious critical analysis of this typology and its appropriateness should be a major focus for the review. Alternatives should be developed and considered.

Student access is an issue for the articulation between junior secondary and senior secondary mathematics courses. Whilst it appears that, through the existence of a range of courses that students are able to make a choice among these, there is, for many students, no real choice. What they can do is constrained by a structured hierarchy of courses (for weak, stronger, even stronger and really strong students). Students can only 'choose' courses for which they have reached some prior standard, so in fact their 'choice' is not really a choice at all. Many students seem to be informed about what mathematics courses 'suit' them (based on their previous success or otherwise), so they are not genuinely being offered a choice. Making more choice available to more students may be possible through a new model for a range of courses that do not constitute a strict hierarchy.

The content of Stage 6 mathematics courses

The development of 'cohesive conceptions of mathematics (that) are associated with deep approaches to learning mathematics' is undermined by excessive content. Hence the extent of the content of any course needs to be carefully considered — less done in more depth is clearly preferable to more content covered superficially. An important matter for consultation is therefore whether current courses contain too much content to encourage deep learning. This will provide a basis for working out optimal amounts of content.

The other factor in reviewing and revising the content Stage 6 mathematics courses is the need to reflect the mathematics of the 21st century. Several broad areas of mathematics would appear to lay claim for inclusion including:

- Statistics;
- Logic;
- Numerical methods; and
- Codes, encryption.

Some other jurisdictions have developed courses with substantial components of some of these areas on which NSW developments could be built; for others there would be an opportunity for NSW to take a leading role.

There are clear tensions between the suggestion that the amount of content be in keeping with deep learning on the one hand, and that 'new' content areas be considered for inclusion. It needs to be accepted that the content of the new Stage 6 courses may need to be substantially different from current offerings.

Teaching, learning and assessing mathematics

The redeveloped Stage 6 courses need to promote coherence between these three components of the education process. It is acknowledged that there is a limit to the influence a syllabus as the intended curriculum can have in the classroom and on practices generally (ie the delivered curriculum). That said, there are models around the country for embedding pedagogies in the curriculum. For senior secondary mathematics the sorts of pedagogies that promote deep learning might include group work, projects and a focus on applications. These and others that might be identified through consultation in NSW may be able to be incorporated in the documentation of the courses.

The extent to which certain pedagogies and approaches can be encouraged or even mandated is really something that can only be decided locally. One mechanism that has been used in other jurisdictions has been to use assessment requirements as the 'lever'. The high stakes associated with end of school assessment makes this an especially effective mechanism. Change through assessment can also be very controversial among the educational and wider community, precisely because of the stakes. It is inevitable, however, that substantial change to pedagogy and formative assessment practices in many classrooms is required to achieve the goal of deep learning of mathematics.

The use of mathematically-able technologies in teaching, learning and assessment has been central to the approaches to curriculum in senior school mathematics in other jurisdictions over the past 10-15 years or more. The case for the use of technology in mathematics — and in particular hand-held devices — has been accepted in jurisdiction after jurisdiction in Australia and internationally (International Baccalaureate, France, Germany, USA Advanced Placement etc.). It would be highly anomalous if NSW does not also take this pathway as the benefits and possibilities for teaching and learning are clear. In particular, pedagogies associated with deep learning such as applications that allow students to develop and work with connections within mathematics and with its uses are more effective when technology takes the 'computational load'. Technology allows exploratory "what if…?" questions that encourage thinking and analysis to be explored more often and more thoroughly. These and other benefits have been well documented. In any case, it would be a strange stance indeed to acknowledge that doing mathematics in the 21st century is very often a technological enterprise yet signal that those same technologies have little or no place in the learning of mathematics at school.

What has emerged has been a recognition that allowing (sometimes even expecting) the use of hand-held technology in the formal assessment processes is an essential ingredient in approaches that effectively exploit the potential of technologies for students' learning of mathematics. This needs to be considered in NSW.

The emergence of widely available Computer Algebra Systems (CAS) on hand-held personal devices is a current issue in this area. CAS capable devices are unquestionably a more fundamental challenge to the nature of school mathematics than the previous generation of 'graphics calculators'. Graphics calculators change *how* the same things are done in mathematics; CAS devices change *what* is and can be done.

In the context of the timing of the review and redevelopment of Stage 6 courses in NSW there would seem to be good reasons to carefully consider the appropriateness of CAS capability in the formal assessment of mathematics. Consistent with the experience with logarithm tables and slide rules, through arithmetic and scientific calculators to graphics calculators, it is not a case of whether this technology will be embraced in school mathematics, but when. Taking this step now would place NSW in the group of early adopters (although certainly not the first) able to exploit this technology in its courses. It would require, and go hand-in-hand with, the sort of radical review of content and pedagogy suggested in this paper.