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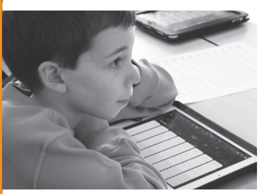


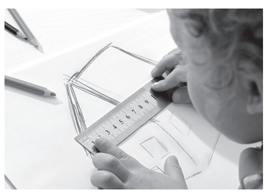

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Lorraine Day <lorraine.day@nd.edu.au>

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Good concrete activity is good mental activity

Andrea McDonough
Australian Catholic University, Vic.
<andrea.mcdonough@acu.edu.au>

Although the use of manipulatives can be helpful for student development of mathematical understanding, it is the crucial role of the teacher in setting up appropriate learning opportunities, asking key questions and encouraging students to explain their thinking that makes learning with manipulatives powerful.

Are you an early years teacher who uses or has children use concrete materials in maths lessons? Do you, like some teachers, wonder about the effectiveness of using concrete materials? If so, perhaps you ask questions like the following:

- For what purposes are concrete materials used in my mathematics classroom?
- Who decides if a student is to use concrete materials and who decides which materials?
- Does the use of concrete materials really stimulate my students' mathematical thinking?

These questions, which might be relevant to any early years mathematics classroom, underpin the telling of classroom stories in this article. The first two stories come directly from my visits to classrooms and then, to extend the discussion, I draw on classroom stories from papers I have read.

Early years mathematics classrooms can be colourful, exciting, and challenging places of learning. Fellow teachers and I have noticed that some students make good decisions about using materials to assist their problem solving, but this is not always the case. These experiences lead me to also ask the following:

- Are concrete materials necessarily helpful for all students in their learning of mathematics?
- Are concrete materials always used as effectively as they might be?

The focus of this article is the use of concrete materials in the early years mathematics classroom, but the issues and questions might apply equally to virtual manipulatives and to use of manipulatives in higher year levels. With the underlying belief that "good concrete activity is good mental activity" (Clements & McMillen, 1996, p. 272), three key messages are discussed. These are that:

- concrete materials can help students focus on key mathematical ideas;
- lessons that incorporate concrete materials can stimulate children's higher order thinking; and
- teachers may need to intervene when students use concrete materials.

Message 1: Concrete materials can help students focus on key mathematical ideas

Recently, as part of my involvement in the Contemporary Teaching and Learning of Mathematics (CTLM) project, I had the opportunity to teach a Prep (Foundation level) class. I wanted to engage the children and challenge them to think about key ideas of mass measurement. In the first activity of the lesson, six 'mystery bags' (paper bags each holding one object and sealed with a peg) were placed in the centre of the circle, resulting in much speculation as to what they might hold. The bags were then passed around, giving each child a chance to hold them (Figure 1). A very special moment occurred when a child who spoke very little English held two bags. Her eyes lit up and she excitedly said "heavy" and "light" to her friend who translated into English for us. I suspect that prior to holding the bags, this child had little sense of the mathematics of the lesson; the holding of the bags was essential for this child (and indeed most probably for all the children). The key mathematical terms she had identified were recorded on the whiteboard.

The children suggested that to check which bag was heaviest we could use balance scales (Figure 2). While the bags were weighed, compared, and ordered, once again there was much discussion. The statement from a child that "the heavy goes down" was recorded on the whiteboard.

Following the weighing and ordering of the bags by mass, we took the objects from the bags. The children discovered that although some were heavier and some lighter they all looked about the same size (volume). I had purposely chosen the objects as seen in Figure 3—a glass paper weight, a foam ball, a tennis ball, a cricket ball, a small apple and a small orange—to expose the children to the key mathematical idea that size (volume) does not necessarily determine mass.

The use of materials in the introduction to this lesson had performed some key purposes:

- Children were engaged in the lesson;
- Key mathematical terms 'heavy' and 'light' were articulated and highlighted;
- Attention was brought to the important measurement idea of comparison;
- Children had the experience of hefting and using balance scales to compare and order masses; and
- The idea that mass is not necessarily related to size (volume) arose in the discussion.

The choice of particular materials, the way they were presented, and the fact that every child had 'hands on' physical experience, helped achieve these purposes. This was the introductory part of a lesson and was followed by children in pairs comparing masses of everyday objects using balance scales. (I had taken in a big mystery bag—containing a mixed collection of shoes—and many children compared these to their own shoes!). The relationship between mass and size was explored by some children in their further investigations. Some focused on comparing and used the related language, and some measured by quantifying using balance scales and informal units available in their classroom. From my experiences of teaching mass at various levels in CTLM schools and through focused research of mass learning in Years 1 and 2 (e.g., McDonough, Cheeseman, & Ferguson, 2013), I believe that a range of further rich experiences with manipulatives and related discussion of the key ideas of mass measurement would be required by these children in Prep and throughout the primary years. However, the use of concrete materials and the related discussion in this one lesson did provide experiences that not only engaged the children physically but also facilitated thinking about key ideas of mass measurement.

Figure 1. Each child had a turn to heft the 'mystery bags'.

Figure 2. The masses of mystery bags were compared by using balance scales.

Figure 3. The objects that were in the mystery bags.

Message 2: Lessons that incorporate concrete materials can stimulate children's higher order thinking

When teaching mathematics to her Prep (Foundation level) class, Ms Prep, a highly effective teacher in the Early Numeracy Research Project (ENRP, McDonough, 2003; McDonough & Clarke, 2003) often used a range of concrete materials to facilitate thinking, used tasks that challenged children, and posed questions that asked them to explain and reason. For this lesson, Ms Prep stated that she wanted to "introduce the children to repeating patterns … They had noticed numbers on the number chart but we hadn't done anything with other patterns at all".

When introducing the lesson, Ms Prep used a variety of stimuli to show different types and uses of patterns. She read a book that included pictures of everyday objects and made reference to patterns on leaves and wings of butterflies. She then showed a collection of objects she had bought in preparation for upcoming family birthdays, including a decorated plastic plate and wrapping paper. When Rory excitedly told the class that the plate had a pattern, Ms Prep asked "How many colours are in the pattern?" George told the class that the colours kept going all the way around the edge of the plate. With the purpose of making the mathematical idea explicit, Ms Prep stated "Around the plate is a repeating pattern". The multi-directional geometric patterns on the sheets of wrapping paper provided extra challenge. At one time Ms Prep asked, "Lara, can you see any pattern there? Is there a repeating pattern?". She also expected children to explain: "There are lots of pretty colours, but is that a repeating pattern? How?" Throughout the discussion Ms Prep brought the students' attention to the mathematics by reinforcing the idea of repeating pattern.

For the main part of the lesson, the children formed a circle on the floor and each made their own pattern using coloured square tiles. For example, one child made a pattern of white, red, white, red, white, red. Another child made the following line of tiles: white, green, yellow, green, yellow, green, white. As the children displayed their patterns Ms Prep challenged the class to think, that is, she held back from commenting on the students' patterns and asked questions including:

- Tell us your pattern. Is that a repeating pattern?
- How many colours make up ….'s pattern?
- Tell us his/her pattern. Is it a repeating pattern?
- Why is it not a repeating pattern?
- Where did it stop being the same?
- How could s/he make that a repeating pattern?
- Has anybody got any suggestions?

The final four questions are examples of those where the teacher took the children beyond description into higher order thinking. They include asking students to show mathematical understanding by justifying.

To conclude the lesson, Ms Prep assessed each student's ability to create a repeating pattern. Choosing anything from a two-to a five-colour repeating pattern, each student used stickers to make a pattern around the top of a plastic cup (the cups were to be used on Grandparents' Day).

To summarise, in this lesson Ms Prep introduced the students to the idea of a repeating pattern through use of a variety of appropriate materials and focused activities. The students engaged not only with the materials but also with the key mathematical idea of the lesson. Ms Prep used open-ended tasks and posed questions to challenge the students to reflect, to explain their mathematical thinking, and to evaluate other students' mathematical ideas. The teacher played a key role in the effectiveness of children's use of the concrete materials for the learning of mathematics. The range of manipulatives provided the context for identifying and discussing repeating patterns and thus was a key element of the lesson, but it was the choice of task and the teacher questioning which stimulated higher order thinking.

Message 3: Teachers may need to intervene when students use concrete materials

A dilemma for teachers can be whether to intervene when children use concrete materials.

There are times when a child has chosen to use certain materials and a suggestion from the teacher to use an alternate representation can confuse the child. Baker (2008) illustrated this with a story from a demonstration lesson in a Year 2 class. Baker started by reading the counting book *One is a Snail, Ten is a Crab* (Sayre & Sayre, 2006). She then asked the children to show what combination of creatures would have 47 feet. One child chose to draw crabs (10 feet) and other creatures to represent the combination. However, one of the observing teachers then intervened and encouraged the child to use bundles of 10 icy pole sticks and loose ones. Baker described that the child was "left in a strange place. What she had been doing made sense to her but she could see it was not valued by the teacher" (p. 36). The child did not make the links between what the intervening teacher described as "pretend feet" (icy pole sticks) and her own drawing. Baker felt that, at that time, the drawing was more meaningful to the child and the teacher's suggestion was disconnected and too abstract.

Importantly, there are also times where intervening and extending a child's thinking beyond the use of materials can be beneficial. This was conveyed very clearly for me in the following classroom story.

Ambrose (2002) warned us that some children use cautious approaches, such as the use of concrete materials, and do not progress to solving problems through abstract approaches. To illustrate how such children work, and as a context for discussing possible teacher responses, Ambrose (p. 17) told about a second grade class where the following problem was posed:

> There were 91 second graders and 37 first graders on the softball team. How many more second graders than first graders on the softball team?

Two children in the same class who had performed similarly on tests of multi-digit addition and base ten knowledge, solved the problem. Their teacher allowed them to choose whatever tools they wanted and expected them to explain their thinking. Paul worked mentally saying "Ninety take away 30 is 60; take away 7 is 53. Add that 1 from the 91, and you have 54". Paul used what might be called an invented mental strategy, a legitimate way to solve the problem. May used blocks, making a train of 91 blocks and another of 37. She placed the trains next to each other and "painstakingly counted the difference between the two trains by ones and obtained the correct answer" (p. 17). The teacher then asked May if she could use tens. May counted the difference between the trains by tens, getting the correct answer again. In some ways May's approach was satisfactory. It was good that she had been exposed to the difference form of subtraction and was able to work with tens when prompted.

But, on the other hand, as Ambrose discussed, May was actually using a concrete procedure that she had been using since her first year of school. When she did not have materials, May would draw tally marks, mostly counting them by ones, thus still using a form of concrete method. May appeared to be working on 'automatic pilot' in choosing to use a modelling approach for solving problems with little if any evidence that her thinking had progressed.

So a dilemma for teachers can be whether or not to intervene when children are using manipulatives. In the case of the classroom story described here it seems that the teacher's knowledge that May had worked with materials in the same way for a long time was a key factor. Perhaps you have children like May in your class and you are asking how you might best help them to move onto solving problems through abstract approaches.

Ambrose (2002) suggested a range of strategies from which I have chosen three that I think can be particularly helpful when working with children like May during maths lessons:

- "When a child solves a problem but cannot explain how she did so, do not prompt her to use manipulatives. Rather than ask her to show you what she did, support her in explaining her thinking. You might ask, 'What number did you start with? What did you do next?' and so on." (p. 20)
- "Encourage children to challenge themselves. Ask, 'Can you think of another way to solve that problem? Can you try working in your head?' "(p. 20)
- "If a child uses manipulatives, ask her to explain what she did without giving her access to the blocks. In other words, push the blocks aside and say, 'Now tell me what you did with the blocks'. This practice will prompt the child to reflect on her actions. By imagining her actions, she will begin to develop mental

pictures of the blocks, on which she might be able to operate in future." (p. 20)

I believe these strategies can be used to encourage children to be 'thinkers' rather than purely 'doers' when using manipulatives. Where a child's long-used method of working with materials is inefficient the child may need to be encouraged by the teacher to think differently about solving the problem. The strategies listed may be good starting points for addressing a need you recognise in children in your class.

Final comments

As I have tried to convey in this article, I believe that manipulatives are of value in the mathematics classroom but are not enough on their own. It is important to focus on student thinking when manipulatives are used. As Waite-Stupiansky and Stupiansky (1998) commented, "busy hands don't necessarily mean busy minds" (p. 85), so teachers need to ask probing questions and stretch children's thinking. Swan and Marshall (2010) perhaps gave a stronger message when they stated that "simply placing one's hands on the manipulative materials will not magically impart mathematical understanding. Without the appropriate discussion and teaching to make the links to the mathematics explicit, the very opposite may be true; children may end up with mathematical misconceptions" (p. 19). The focus on thinking as well as physically doing suggests that the role of the teacher is essential to the effective use of materials. Indeed, Baroody (1989, p. 4) suggested that teachers ask:

> Is the manipulative used in such a way that it requires reflection or thought on the part of my students?

As illustrated in this article, manipulatives do not stand alone as an issue in the teaching of mathematics. The appropriateness of the tasks, the nature of the questions posed by teachers, encouragement for children to explain thinking, and explicit reference to mathematical terminology are all related and important considerations. Extending children's own thinking through having them evaluate others' mathematical thinking/ideas, and taking steps at an appropriate time to encourage children to advance beyond reliance on materials are further key considerations for teachers. The aim is for children to come to solve problems through abstract approaches. By taking account of these related issues, teachers play a key role in creating for children good concrete activity that is good mental activity.

The classroom stories I have included in this article are all from early years' classrooms. However, like the majority of teachers surveyed by Swan and Marshall (2010), I support the use of manipulatives at all levels of the primary school and for all areas of mathematics. I suggest the three key messages in this article have relevance in mathematics classes throughout the primary school. Perhaps the messages resonate with issues or questions about the use of manipulatives that have arisen in your maths classes or have been identified by other teachers at your school.

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