Title of chapter/article	More Than Mathematics: Developing Effective Problem Solvers
Author(s)	Amie Albrecht
Copyright owner	The Australian Association of Mathematics Teachers Inc.
Published in	Mathematics: Learn, Lead, Link. Proceedings of the 25th Biennial Conference of the Australian Association of Mathematics Teachers
Year of publication	2015
Page range	8–19
ISBN/ISSN	978-1-875900-79-4

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MORE THAN MATHEMATICS: DEVELOPING EFFECTIVE PROBLEM SOLVERS

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Complex, loosely-defined problems encountered in both the workplace and everyday life demand more than technical proficiency in mathematics. They also require broader capabilities including formulating problems, devising and implementing solution approaches, creativity, teamwork, project management, and communication skills. Significantly, these skills are often needed for any challenging mathematical problem — independent of whether it originates in the 'real world' or not.

This paper explores two questions. How do we meaningfully prepare our students with these skills in a mathematical setting? How can we develop and broaden their abilities and confidence in posing and solving mathematical problems? In this discussion I will draw on my experiences in working as an industrial mathematician, training workplace-ready students, and teaching a new course specifically designed to build mathematical thinking and problem-solving skills in pre-service teachers through games and puzzles.

Introduction

It is a privilege to give this lecture, which celebrates and honours Hanna Neumann. Biographical accounts show her to be a remarkable and inspirational person, who made significant contributions to both mathematics and mathematics education throughout her distinguished career, first in Britain (1938–63) and then Australia (1963–71). (I wholeheartedly recommend the biography by Newman and Wall (1974) which can be read online.) Among her many accomplishments, Hanna was the first woman appointed as a professor of mathematics in an Australian university (at the Australian National University in 1964). Hanna was also instrumental in the formation of the Australian Association of Mathematics Teachers (AAMT). The great esteem in which both communities still hold Hanna is evident, with the AAMT and the Australian Mathematical Society including named lectures at their society meetings in her honour.

Hanna strove to show that "doing and thinking about mathematics can be joyous human activities" and to make mathematics accessible and valued—for both its intrinsic beauty and its practical application—by all (Newman and Wall, 1974). She believed that "the community had to be educated to create a more favourable climate (one in which mathematics is not feared) for the learning of mathematics—especially among girls."

As Hanna herself said to a joint meeting of the Mathematical Association of South Australia and the Australian Mathematical Society in 1971 (Neumann, 1973):

> Speaking about teaching to trained teachers makes me very conscious of our, that is the university teachers', lack of training — I wish I had not got to say this! But this is by-the-way; perhaps I have been sufficiently long occupied, indeed at times pre-occupied, with the teaching side of my job that this undertaking is not entirely ludicrous.

With this firmly in mind I will discuss how we can develop, strengthen and broaden our students' mathematical skills and deepen their appreciation of the power of mathematics by linking to both engaging real-world problems and recreational puzzles. I will start with inspiration from my mathematical area of Operations Research.

Operations research: The science of making smarter decisions

Operations research (OR) is the scientific approach to decision making. In OR, there is not a single technique that can be used to solve all mathematical problems that arise. Rather, an OR practitioner like myself will select the most appropriate technique from across the mathematical sciences, including mathematical modelling, probability and statistics, optimisation, stochastic processes, simulation and game theory.

Operations research is a very practical discipline. From its origins in British military applications during World War I, OR has expanded to find significant implementation in many other areas. For example, some of the special issues of the journal *Interfaces*, which emphasises the impact of OR in organisations, have focused on:

- energy industry
- freight transportation and logistics
- mining
- humanitarian applications
- sports analytics and scheduling
- health care
- military applications
- finance
- marketing
- forest products industry.

This short list illustrates the breadth of OR; it would be easy to fill the page with more examples. (Try www.LearnAboutOR.co.uk for teacher resources of OR in everyday life.)

While mathematical analysis is at the heart of solving OR problems, Operations Research is more than mathematics. Taha (2011) comments that:

> OR is both a science and an art. It is a science by virtue of the mathematical techniques it embodies, and it is an art because the success of the phases leading to the solution of the mathematical model depends largely on the creativity and experience of the OR team.

Broadly, the five phases of implementing Operations Research in practice are:

1. *Defining the problem* and the objective(s). This is especially challenging for complex or ill-defined real-world problems. It usually entails observing the current system and comprehending a lot of domain-specific knowledge.
2. *Constructing a mathematical model*, including identifying any assumptions and simplifications.

3. *Solving the mathematical model*, which might require mathematical techniques ranging from elementary to sophisticated, or computer-based approaches for mathematically intractable problems.
4. *Validating the model*, typically by using historical data, covering a variety of situations, to determine if the model is an accurate representation of reality. A 'common sense' check is also recommended as is careful investigation of any 'surprising' solutions. Also 'does the model solve the original problem?'.
5. *Implementing the solution or recommendations*, which includes translating and summarising techniques, analysis, outcomes and findings into appropriate and usually non-technical language for the 'customer'.

This methodology is rarely linear and it is likely that phases will be revisited as the process of finding a solution evolves.

Industry problems[1] are nearly always too complex for one person to work on. In addition, the multi-disciplinary nature almost always requires teamwork with professionals in other fields.

Case study: Keeping trains on track and on time

My earliest experience in tackling industrial OR problems was though my PhD research (2002–09), which investigated operationally feasible methods to produce integrated train timetables and track maintenance schedules so that, when evaluated according to key performance criteria, the overall schedule is the best possible.

A train timetable specifies a path, with timing information, for each train through the network. A train timetable is represented graphically with a 'string line diagram'. Figure 1 is one of the earliest train timetables (1885), from Paris to Lyon, and shows time on the horizontal axis and distance on the vertical axis. Existing methods schedule track maintenance once the train timetable has been determined, which almost certainly produces sub-optimal solutions.

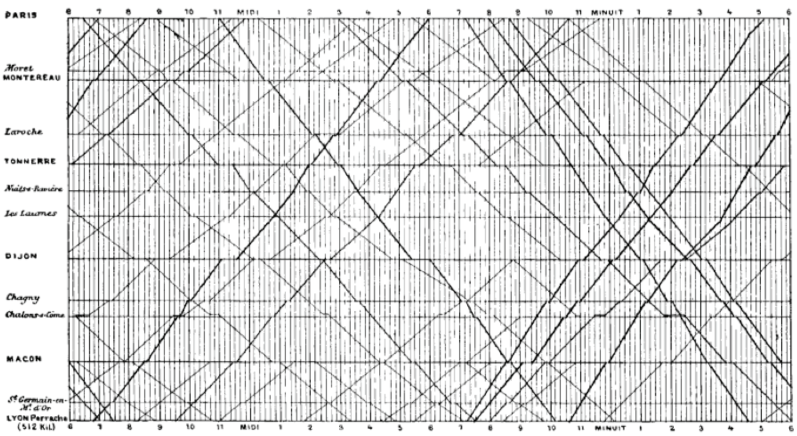

Figure 1. String line diagram (train timetable) from Paris to Lyon (1885).

After carefully observing the manual creation of timetables in train control centres, talking with train controllers and analysing historical data, I was able to formulate an integer linear programming model to mathematically describe the scheduling process. I

[1] The term 'industry problems' is used to describe problems arising from outside of academia, including from business, industry, government and non-corporate entities.

painstakingly applied a sophisticated mathematical technique to find provably optimal solutions but found that it was not operationally feasible for realistic-sized problems. I then devised a fast probabilistic computer algorithm to efficiently generate thousands of alternative schedules in minutes from which good solutions could be selected. I benchmarked by simulating and comparing with the existing manual process. I wrote thousands of lines of computer code. I gave presentations to both industry and academia, and interviews to media. I wrote reports targeted at industry, mathematical papers for academia, and ultimately a PhD thesis. My work contributed to a larger project with an expected value of $7.6 million (2006 report) to the Australian rail industry and which was commercialised into industry-ready software. However, the research had only isolated uptake in industry, partly because we neglected to fully appreciate both human behaviour and the organisational shift required to implement our work. As Taha (2011) notes, "... while mathematical modelling is a cornerstone of OR, unquantifiable factors (such as human behaviour) must also be accounted for."

Despite the somewhat unsatisfying conclusion, this case study demonstrates the breadth of skills needed to meaningfully tackle challenging mathematical problems.

Maths in careers: Enhancing job prospects

A recent report commissioned by the Office of the Chief Scientist, *Australia's STEM workforce: a survey of employers,* and released on 29 April 2015, found that the top five skills of STEM[2] employees that are of highest importance to employers are: active learning (ability to learn on the job), critical thinking, complex problem-solving, creative problem-solving, and interpersonal skills. Respondents also listed skills important in their workplace in addition to those in the survey. The report found that "overwhelmingly, communication skills were most predominantly mentioned, followed by writing, project management, marketing, financial and leadership skills." When recruiting workers, academic results were considered of far less importance than interpersonal skills (with nearly half of respondents rating this as very important).

While the importance of technical skills (which was not specifically included in the 'skills and attributes' survey) was considered fundamental, more than 80% of employers agreed that people with STEM qualifications are valuable to the workplace even when their major field of study is not a prerequisite for their role. STEM employees were considered among the most innovative and adaptable in workplaces.

The study of mathematics—to whatever level—builds skills transferable to other disciplines and occupations. For example, studying mathematics requires the ability to think clearly, pay attention to detail, follow complex reasoning and construct logical arguments—valuable skills to have in many contexts! (See the extensive list from the University of Warwick (2011) of transferable skills obtained through study of mathematics.)

Careers in maths: The hidden mathematician

Job advertisements for those with mathematics and statistics qualifications rarely list it in the position titles. Mathematically-trained professionals work across a diverse range of industry, business and government sectors. The yearly 'Maths Ad(d)s' publication[3] by

[2] Science, Technology, Engineering, and Mathematics.
[3] Maths Ad(d)s booklets can be ordered from AMSI (careers.amsi.org.au/mathsadds) or downloaded as a free PDF.

the Australian Mathematical Sciences Institute (AMSI) gathers together a sample of job advertisements for careers requiring training in mathematics and statistics. Recent Maths Ad(d)s include: bioinformatician, remuneration analyst, software programmer, data scientist, epidemiologist, energy forecaster, credit risk modeller, market researcher, meteorologist, behavioural scientist, operations manager, quantitative analyst, teacher, and groundwater modeller. For more than 101 careers in mathematics, see the book with the same name published by the Mathematical Association of America (Sterrett, 2014).

Training our future mathematically-capable professionals

So, how do we meaningfully prepare our students to be mathematically capable and workplace ready? I offer two models with which I have had experience: one at tertiary level and one at secondary level.

The UniSA Mathematics Clinic

The Mathematics Clinic was started in 1973 by the Mathematics Department at Harvey Mudd College (HMC) in Claremont, California, to train industry-bound mathematics graduates. It is a sister program to the Engineering Clinic established in the early 1960s by the Engineering Department at HMC and was considered to be like medical clinics which provide interns with hands-on real-world experience in a supervised environment (Borrelli, 2010). The Clinic solicits real-world open-ended problems (called projects) from companies and government agencies. Over 150 Mathematics Clinic projects have been conducted at HMC[4].

The Mathematics Clinic[5] program at the University of South Australia (UniSA) is based on the HMC model and is the only one of its type in Australia. It is a year-long team-based sponsored project undertaken by final-year mathematics undergraduates. It provides a rigorous research experience in tackling mathematics problems sourced from industry. It is primarily an educational program, although it also delivers products (e.g. mathematical algorithms, reports, presentations, analyses, software).

In 2014 the UniSA Mathematics Clinic team developed mathematical techniques and prototype software to aid the Australian Army in determining both the number and type of new land combat vehicles required to replace the existing fleet. The Clinic project contributed to Land 400—the largest project the Army has ever undertaken. The Defence Science and Technology Organisation (DSTO) was the project sponsor.

A UniSA Clinic team typically consists of four students, an Academic Supervisor, an Academic Consultant who provides specialised mathematical expertise (which in 2014 was me), and a sponsor Liaison to monitor progress and provide domain-specific expertise. A student is appointed as the Project Manager; others take on leadership roles throughout the year, typically to lead the completion of a project deliverable.

The first deliverable is the Work Statement—the formal agreement between the team and the sponsor to accomplish certain tasks and produce certain products. It drives the process of defining the problem, as well as planning the management of and schedule

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[4] www.math.hmc.edu/clinic/

[5] The current UniSA Mathematics Clinic Director is Associate Professor Lesley Ward. More details at www.unisa.edu.au/IT-Engineering-and-the-Environment/Information-Technology-and-Mathematical-Sciences/Maths-Clinic/

for the entire project. Writing the Work Statement is often the first meaningful experience that students have in translating a complex real-world problem into a form that can be tackled mathematically. One particular challenge for the 2014 Clinic team was to identify how they could add value to Land 400 within the constraints of a year-long student project. The team decided to analyse the number of vehicles required for three specific types of missions (screening an object from the enemy, searching for a target within an area, and clearing an urban area of enemies). They also reviewed and compared several multi-criteria decision-making methods that could be used to select which vehicles to purchase, as different types of vehicles are deemed more (or less) suitable depending on which characteristic they are judged by.

Preparing the Work Statement requires intensive discussions between the liaison and the team, and usually a site visit. Our students discovered the acronym-rich world of Defence and by the end of the project became conversant in the jargon—a large shift from the start of the year!

The mathematical effort required to tackle the project is initiated and undertaken by the students with guidance from the academic advisors. The students meet regularly for work sessions and to divide up the tasks. The 2014 Clinic team needed to juggle four sub-projects and formed smaller work-teams for some tasks. Regular meetings are also held with the sponsor liaison and academic staff to provide updates and to seek clarification or guidance. Students are coached in chairing meetings, preparing agendas, managing projects, interacting with the sponsor, building and maintaining effective teams, dealing with inevitable problems and issues, and re-negotiating the scope of the project if needed.

Clinic teams give a formal mid-project written report and presentation. The project culminates with a professional-quality final report, presentation to the sponsor in their workplace, and delivery of project outputs. Clinic teams often write academic papers and give conference presentations. Students are trained in developing these skills, and in delivering and targeting a technical message to be accessible by different audiences.

The Mathematics Clinic is an enriching experience that builds not only professional skills, but student self-confidence. It demonstrates the link between mathematical knowledge and application, and provides exposure to a potential employer. It was a real joy to watch the 2014 Clinic students blossom into self-directed, capable, industry-ready mathematics graduates.

School to work

The 2010 SACE[6] 'School to Work' Mathematics and Science programs were designed to partner schools with industry or universities to devise innovative ways to make maths and science interesting and relevant, ensure students understand the value of maths and science, and help to connect students with maths and science career paths.

I was involved in the 'Developing Mathematicians' project[7] at St Michael's College in Adelaide which provided a structured year-long program to engage Year 10 students in genuine, relevant and challenging mathematical research experiences and build transferable skills in preparation for both the SACE Research Project and their future

[6] South Australian Certificate of Education

[7] The project was led at St Michael's College by Dr Pauline Carter, Mark Winston and Carmen Swan. Dr Amie Albrecht was the supporting mathematician.

careers. The project deliberately took on a gendered focus by bringing together two single-gendered classes (25 boys and 24 girls) and including a female mathematician.

In the first two terms, students were guided through the train planning process, beginning with demographic analysis of the travel demand, designing train services to meet demand, and constructing detailed operational train timetables. Sessions were loosely directed with a workbook and centred around group discussion and a hands-on approach. Teachers, a university mathematician and old scholars assisted as ‘guides on the side’. Students experienced the complexities of authentic mathematical work.

Having developed skills and confidence, in the second half of the year students worked on problems from a local steel industry in a model that parallels the Mathematics Clinic. The teachers and mathematician did some pre-work to develop simplified realistic scenarios. Students designed mathematical algorithms and spreadsheets to determine material requirements for orders in fencing and roofing. They visited the site and eventually made final presentations to the industry liaison.

A large part of the success of the project was the authenticity brought by engaging students and teachers with mathematicians and industry clients. Several of the students continued working on the problems beyond the end of the project. We observed that female students developed a more confident and resilient approach to learning mathematics, were more receptive to tackling a challenge, and less reliant on teacher direction. Students’ understanding of mathematical careers were broadened.

The two models I have described are significant undertakings, but hopefully they provide inspiration for bringing elements of the experience into the classroom. Authentic problems can be obtained from many places; a teacher at St Michael’s College had a connection with the steel company. Enlist old scholars or perhaps link with the Mathematicians in Schools[8] initiative. Peruse the archive of stimulating projects from the Moody’s Mega Math Challenge[9] in the US, which is accompanied by an excellent handbook describing the modelling process (there are other similar competitions, both locally and globally). Draw on problems that students encounter in their own lives. I also encourage you to look for engaging questions in your own surroundings; Dan Meyer’s 101questions site[10] has an ever-growing collection of contributed photos and videos that provoke questions and are suitable for projects.

Puzzles and other pastimes

Engaging problems are not confined to the ‘real world’. The ‘Cheryl’s birthday’ puzzle recently captivated the attention of many, particularly via social media. It is as follows[11]:

> Albert and Bernard just became friends with Cheryl and they want to know when her birthday is. She gives them a list of 10 possible dates:

15 May	16 May	19 May
17 June	18 June	
14 July	16 July	
14 August	15 August	17 August

> Cheryl then whispers to Albert which month her birthday is in, and whispers to Bernard which day her birthday is on.
> Albert: I don't know when Cheryl's birthday is, but I can be certain that Bernard does not know either.

[8] www.mathematiciansinschools.edu.au
[9] m3challenge.siam.org
[10] www.101qs.com
[11] This is Rob Eastaway’s version, reworded for clarity. Source: www.robeastaway.com/blog.

Bernard: At first I didn't know when Cheryl's birthday is, but now I do know.
Albert: Then I also know when Cheryl's birthday is.
So when is Cheryl's birthday?

This scenario is highly unlikely to occur in real life—your students will probably tell you just to 'friend' Cheryl on Facebook and then you'll be notified when her birthday[12] comes along!—but puzzles such as these can be as stimulating and challenging as those that arise from real-world contexts. Like many people, I can be engrossed solving Sudokus, struggling with a Rubik's Cube or a logic puzzle like 'Cheryl's birthday', sliding tiles in Tetris, or suspecting that the three utilities problem is impossible. Many board and electronic games require logical thinking or strategising—and are also fun!

Companies like Google and Microsoft famously ask potential employees to solve brainteasers and maths riddles for a good reason; these puzzles require the types of 'thinking skills' that employers are looking for. So, can we intentionally develop these mathematical thinking skills in our students by using puzzles?

Puzzle-based learning

A few years ago, I was inspired by the puzzle-based learning approach of Michalewicz and Michalewicz (2008) which "focuses on getting students to think about framing and solving unstructured problems (those that are not encountered at the end of some textbook chapter)" with the purpose of increasing "the student's mathematical awareness and problem-solving skills by solving a variety of puzzles and reflecting on their solution processes" (Meyer et al., 2014, p. ix). As the authors point out, the educational use of puzzles is not new, with a long history preceding that of their work and 20th century champions such as Gyorgy Polya and Martin Gardner. However, the puzzle-based learning course (currently taught at the University of Adelaide, where Z. Michalewicz is Emeritus Professor in Computer Science) was the first structured course using puzzles that I had seen, with a syllabus organised around problem-solving techniques and various mathematical topics. Students in the course at the University of Adelaide are assumed to have knowledge of SACE Stage 2 Mathematical Studies but it is not a prerequisite. Details of how the course runs are in Falkner et al. (2010).

Training our future teachers

At about the same time that I discovered puzzle-based learning, I was reflecting on the way in which we at UniSA teach mathematical content (as distinct from pedagogy) to our pre-service primary and middle teachers who choose mathematics as a specialisation, and whether it exemplifies the types of experiences I want them to have.

The mathematical prerequisite for entry into our primary and middle teaching degree is one semester of Year 11 mathematics (as a result of completing the SACE, although some students have done more mathematics). Despite having chosen it as an area of specialisation, many students describe themselves as lacking confidence or experiencing anxiety when doing mathematics—especially when tackling problems that are unfamiliar or challenging.

[12] Cheryl's birthday is July 16.

The importance of being stuck

In my experience, most students associate making mistakes and being stuck with an inability to do mathematics. In contrast, professional mathematicians spend much of their time quite comfortably feeling unsure of the next step. This is often hidden from students as Mason et al. (2010, p. ix) describe:

> Elegant solutions such as are found in most mathematics texts rarely spring fully formed from someone's brain. They are more often arrived at after a long and tortuous period of thinking and not thinking, with much modification and changing of understanding along the way, but most beginners do not realise this. ... Elegance can come later.

I want my students to feel stuck. I am glad when they are. Research about the brain shows that "when students make a mistake in maths, their brains grow, synapses fire, and connections are made. This finding tells us that we want students to make mistakes in maths class and that students should not view mistakes as learning failures but as learning achievements" (Boaler, 2015, p. xix). Encouraging our students to develop a growth mindset ('the more they work the smarter they will get') rather than a fixed mindset ('some people are naturally good at maths and some are not') is key in their mathematical development. It is important though, that this struggle is productive. We can assist by equipping our students with an awareness of the different phases and processes that take part in mathematical thinking, as well as acknowledging and discussing their emotional responses—both negative and positive.

A course in 'Developing Mathematical Thinking'

In 2014 at UniSA we offered a new elective course to pre-service primary and middle teachers called 'Developing Mathematical Thinking'. In its design, I drew together many of the elements I have already elaborated on—stimulating curiosity, promoting mathematical communication, developing confidence, increasing problem-solving skills—and an intention to use mainly puzzles, some of which are 'hands on'. Of particular interest were 'low-threshold, high-ceiling' activities "which pretty well everyone in the group can begin, and then work on at their own level of engagement, but which [have] lots of possibilities for the participants to do much more challenging mathematics" (McClure, 2011). (The NRICH Project[13] is a good source of activities.)

To articulate mathematical processes and phases of work, the course rests heavily on the framework provided in the book *Thinking mathematically* (Mason et al., 2010). Each week is dedicated to one or more themes. Students work, mostly in groups and at their own pace, on problems that require a range of mathematical techniques but that are purposely selected by me to reinforce the week's themes. I will occasionally teach a specific mathematical concept to the whole class, but it is more likely that students will teach each other as needed. The weekly themes include:

- *Specialising* (trying specific examples to get to grips with a problem) and *generalising* (detecting a pattern that holds for a wide class of specific cases).
- *The Entry phase* in which we work to understand the problem.
  - *What do I know* (from the question or from experience)?
  - *What do I want?* To find an answer? To prove something?

[13] nrich.maths.org

   - *What can I introduce?* Definitions, notation, diagrams, tables, physical models, other ways to systematically record work.
   - Framing our own questions (for example, using 101qs.com as starters).
- The role of intuition.
- Strategies for 'getting unstuck'.
- Working *systematically* to detect patterns or expose cases for which a theory might not hold.
- *The Attack phase* in which we try to solve the problem.
   - Making conjectures.
   - Justifying and convincing—yourself, a friend, an enemy.
   - What it means to prove something and why we need proof. (Detecting a pattern that holds for a few cases is not enough!)
- *The Review phase* which includes checking and extending work.
- Writing to record your thinking. Writing to explain your work to someone else.

We discuss these themes in class, and students reflect on them in their formative and summative assessment work. Although I could elaborate on many elements of this course, I will conclude by describing two—a small weekly 'starter' and the student-selected major projects.

Weekly starter: Exposing multiple problem-solving approaches

To encourage exploration of multiple valid ways to solve a problem and to promote mathematical discussion, each week starts with a 'visual pattern', usually drawn from Fawn Nguyen's site www.visualpatterns.org. An example is shown in Figure 2.

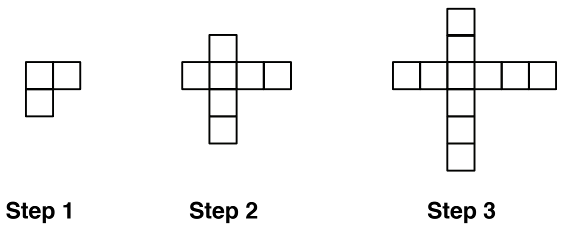
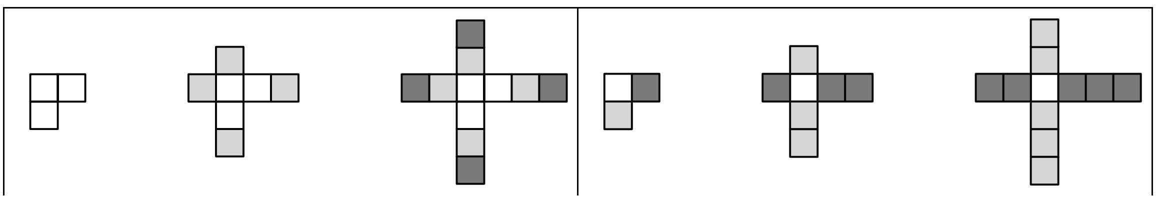


Figure 2. Visual pattern.

Students work individually on the following questions for about five minutes.

- Draw the figure in the next step.
- How many squares are in the figure you just drew?
- How many squares are in Step 43?
- What is the equation for this pattern? (How many squares in Step *n*?)

Students then discuss with their neighbours to help improve their approach. We then gather together many different approaches as a class, with students sharing their reasoning. Examples are shown in Figure 3. Each equation can be algebraically shown to be equivalent to $4n - 1$. We equally celebrate incomplete and complete approaches.



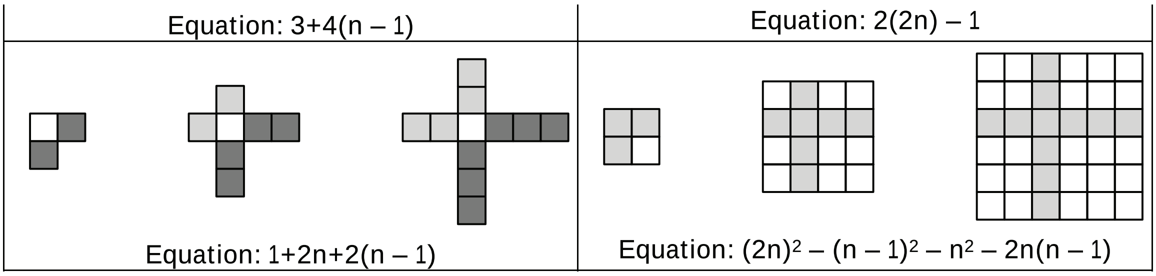


Figure 3. Four possible approaches to the equation for the pattern in Figure 1. The shading is intended to convey the way in which the pattern grows or the figure can be decomposed.

Student projects

The project is a major body of work, drawing together concepts from the entire course. It is an in-depth investigation by an individual student or pair of students on a topic of their own choosing. The project comprises several structured activities over the semester which together are worth more than half of a student's final grade.

Students discuss their choice of topic with me in Week 5. I provide a range of possible ideas and students can also propose their own. Here is a selection from 2014:

- Grid-based logic puzzles like Sudoku. How many different puzzles are there for a given grid size? How are they created to ensure only one solution? What are the best solving strategies?
- Board and card games like the Game of SET in which the goal is to form SETs where each attribute is 'all-same' or 'all-different' across three cards. One student investigated the maximum number of cards that do not contain a SET.
- Mathematical art like fractals, and algorithmic drawing in which artwork is created by following a sequence of rules (e.g. turn 90 degrees clockwise and draw a line with length from a pre-specified list). What are the rules that determine if the drawing is a closed loop or has other interesting properties?
- Physical puzzles like the Tower of Hanoi. What is the minimum number of moves required for *n* disks? How does this change if the number of towers changes? What if the rules for placing disks on towers are altered?
- Pattern and algebra-based questions like the Leap Frog Puzzle or extending the question 'How many squares on a chessboard?' to triangular grids.

In Week 8 students obtain feedback on a draft report from me and via peer review (which is supported in various ways). The purpose of the draft is to help develop and distil ideas; uncover any flaws in the problem-solving work; practice mathematical writing; and enable mid-course corrections, if needed. It encourages the process of revising existing work and incorporating new work. Students meet again with me in Week 10 to discuss their draft. The polished final report is submitted in Week 14 and accompanied by a final 10-minute oral presentation to the class.

Students are trained in giving mathematical talks through weekly smaller talks designed to progressively build skills and confidence. Students start by 'presenting' at their desks to a friend and with the support of their notes. They gradually advance to presenting at the whiteboard, to classmates they might not know, with props and carefully chosen examples. The length of the talks increases incrementally. By the time students give their final presentations, they are experienced at delivering quality talks.

While it is too early to measure the impact that this course has on their further mathematics study, students' reflections show the effects they most valued: "I was encouraged to use a variety of strategies to complete tasks and extend my thinking to a higher level", "I was constantly challenged but never felt I was out of my depth", "The course has developed my math knowledge and expanded it to approach problems differently" and "Working in groups not only broadened my knowledge but it also made me understand some other ways in which different people think".

For me, helping our prospective mathematics teachers develop into enthusiastic, confident and capable problem solvers gives me both great satisfaction and optimism for the impact they will have on their own students in the future.

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