Average task: Possible answers

http://topdrawer.aamt.edu.au/Statistics/Assessment/Assessment-tasks/Critical-thinking-about-average

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> To get the average number of children per family in a town, a teacher counted the total number of children. She then divided the number by 50, the total number of families. The average number of children per family was 2.2.
>
> Which of these is certain to be true?
>
> (a) Half of the families in the town have more than 2 children.
>
> (b) More families in the town have 3 children than have 2 children.
>
> (c) There is a total of 110 children in the town.
>
> (d) There are 2.2 children in the town for every adult.
>
> (e) The most common number of children in a family is 2.
>
> (f) None of the above.

Assuming that an alternative is selected by agreement with its content, rather than by guessing, it is possible to suggest levels of understanding associated with the choices. At the lowest level are the responses (a) and (b), exhibiting a single idea about average that appears influenced by the average measure of 2.2, but which does not *account* for how the average of 2.2 is a central measure. The first part of each response shows a hint of a modal idea but the inconsistency of the rest of the statement is not recognised in relation to the stated average of 2.2 in the core of the question.

Alternatives judged to be at the next level are potentially more complex in recognising the average as a central or typical measure of the data, either a rate (d) or a modal value (e).

Responses that select (f) possibly reject the other responses as they do not satisfy the student's functional definition (e.g. a student using a definition of average as 'middle value' might be expected to give this response), yet fail to consider option (c) by testing 110/50, which indeed yields an average of 2.2. Responses selecting (c) either test the algorithm to verify 110/50 = 2.2, or else reverse the algorithm by multiplying 2.2 × 50 = 110, indicating the student can relate the algorithm to the concrete situation.

Extension

This item could be used as an open-ended enquiry where students are asked to display justifications for their rejection of the incorrect answers, as well as their justification for the choice of the correct answer (i.e. more than just 'choosing' it).

> Provide a mathematical justification for your choice of the correct answer.
>
> Then provide two answers demonstrating why the other alternatives are not certain to be true (i.e., one example where the claim is true and one where it is false).

There are many numerical examples that would make statements for (a), (b) and (e) true and many that would make them false. Always, however, (c) is true and the language in (d) is quite ambiguous. The two examples here will work as either true or false for (a), (b) and (c) but should NOT be regarded as unique 'correct' answers.

Example 1

The numbers in the table below satisfy the condition of 50 families with an average of 2.2 children.

Number of children per family	Number of families	Total number of children
3	25	75
2	10	20
1	15	15
	50	**110**

Example 2

The numbers in the table below satisfy the condition of 50 families with an average of 2.2 children.

Number of children per family	Number of families	Total number of children
3	10	30
2	40	80
	50	**110**

Now consider the alternative responses for the problem.

For (a), we require that half of the families in the town have more than 2 children. This is satisfied in Example 1 where 25 (exactly half) of the families have 3 children. Hence the claim about 'more than 2 children' can be true in some cases where there is an average of 2.2 children. However, looking at Example 2, here is a case with 2.2 children where half of the families do NOT have 'more than 2 children.' Hence the claim about 'more than 2 children' can sometimes be true and sometimes false.

For (b), we require more families in the town to have 3 children than 2 children. Again Example 1 shows that this statement may be true with 2.2 children (because 25 families have 3 children and 10 families have 2 children) but Example 2 shows that it need not be true with 2.2 children (because 10 families have 3 children and 40 families have 2 children).

For (e), we require that the most common number of children in a family is 2. This time Example 2 provides a case where the statement is true with 2.2 children (i.e. 40 families – the most common result – have 2 children), whereas Example 1 shows that it need not be true with 2.2 children (i.e. here only 10 families have 2 children whereas the most common number is 3 for 25 families).

For (d), the statement is ambiguous because it says 'every adult' not 'every family'. If each of the 50 families had 2 adult parents, then the average per adult would be 1.1 children. If every family in the town had just 1 adult in the family then the statement would be true. But also there may be adults in the town who do not have children and have not been counted as families. We need to know more about how 'family' was defined. Hence the statement may or may not be true.

A task such as this is a good challenge for students' critical thinking and argument. Students in a class may produce many examples and counter-examples and these could be checked by other members of the class.