Paperclip investigation: Work sample

http://topdrawer.aamt.edu.au/Statistics/Good-teaching/Making-informal-inferences/Single-measurement-variables/Paperclips

Introduction

In the world there are some populations that are too large just to count simply. So how do we count them? There are various methods such as tag-release-recapture, sampling and weighing. For example, if you wanted to find the population of pandas in China using the tag-release-recapture method you would complete the following steps:

1. First, you would capture a number of pandas, say 20, then tag them.
2. You would release them back into the wild.
3. You would capture more than the amount you tagged, say 100, and see how many of the tagged are in this population.

With this information you could approximately estimate the population using this method:

> If, when you recaptured the pandas, 5 were tagged it means that 5 in every 100 were tagged.
>
> Therefore if the sample of 100 was representative of the entire population, the ratio of tagged Pandas will be the same in the population (20: ?) as it is in the sample (5:100).
>
> 5 : 100 = 20 : ? The population would be approximately 400.

A container is labelled to contain 1000 coloured paper clips. It is possible that some of the paper clips have been used. The task is to first find out the total number of paper clips in the container and also determine how many of each colour there are.

In an effort to estimate the total number of paper clips in the container, the tag- release-recapture method explained above can be used.

The estimations of colour will be determined by sampling and are based on the total number of paper clips present.

Analysis

Tag–release–recapture estimate

The tag–release–recapture method was used to work out the total number of paper clips.

Instead of tagging paper clips, silver paper clips were added. 100 silver paper clips were added to the population. It was decided to add this many because the population was expected to be around 1000 and if only a small number were added they may not be part of the sample. 100 (>10%) was considered a significant number. If there were still 1000 paper clips in the container originally, there were now 1100 as 100 silver clips have been added (1000 +100). A sample of 110 would be expected to contain 10 silver paper clips. Five samples of 110 clips were taken and recorded.

Sample 1

9 silver paper clips were taken out of 110.

Prediction based on sample 1

Ratio in sample = Ratio in Population

$$9:110 = 100:P$$
$$\frac{9}{110} = \frac{100}{P}$$
$$9P = 11000$$
$$P = \frac{11000}{9}$$
$$P \approx 1222$$

Approximately 1222 paper clips.

Sample 2

10 silver paper clips were taken out of 110

Prediction based on sample 2

Ratio in sample = Ratio in Population

$$10:100 = 100:P$$
$$\frac{10}{110} = \frac{100}{P}$$
$$10P = 11000$$
$$P = \frac{11000}{10}$$
$$P = 1100$$

Approximately 1100 paper clips.

Sample 3

18 silver paper clips were taken out of 110.

Prediction after 1 sample

Ratio in sample = Ratio in Population

$$18:110 = 100:P$$
$$\frac{18}{110} = \frac{100}{P}$$
$$18P = 11000$$
$$P = \frac{11000}{18}$$
$$P \approx 611$$

Approximately 611 paper clips.

Sample 4

16 silver paper clips were taken out of 110 (by a similar calculation).

Approximately 688 paper clips.

Sample 5

12 silver paper clips were taken out of 110 (by a similar calculation).

Approximately 917 paper clips.

There was considerable variation in the results for each sample. The accuracy could be improved by taking more samples or larger samples.

To gain a better estimate based on the data collected already, the average number of silver clips in a sample of 110 was calculated and this was used to estimate the total number of paper clips in the container.

The mean no. of silver clips for the 5 samples:

$$x = \frac{9+10+18-16+12}{5} = 13$$

Based on 13 silver clips in a sample of 110,

$$\frac{13}{110} = \frac{100}{P}$$
$$13P = 11000$$
$$P = \frac{11000}{13}$$
$$P \approx 846$$

Based on these calculations it is estimated that there are 846 paper clips in the container now, which means there were **746** before the 100 silver clips were added.

Weighing estimate

Weight could also be used to predict the total number of paper clips. The weight of a paper clip was approximately 0.385 of a gram. All colours were weighed to check that they were not different weights.

These are the weights:

WHITE: 0.40 g	PINK: 0.38 g	RED: 0.38 g	YELLOW: 0.39 g
BLUE: 0.38 g	BLACK: 0.39 g	GREEN: 0.38 g	ORANGE: 0.38 g

On the basis of these results we concluded that the colour of clip did not affect its weight but that there was some variation in the weight of each clip regardless of its colour.

The container weighed 369 g with the paper clips and 56 g without the paper clips.

Therefore altogether the paper clips weighed 313 g.

$$\text{Total number of paperclips} = \frac{313}{0.385}$$
$$= 813 \text{ paperclips}$$

According to tag–release–recapture method there are 746 paper clips and according to weighing there are 813 paper clips.

I think that weighing is more accurate because with the tag-release-recapture method there is some chance involved. There will always be variation in random samples. Although it is unlikely you could get anywhere from 0–100 silver paper clips in a random sample of 110. Weighing is very accurate because although there will be inaccuracies in measurement and small variation in the weight of each clip, there is no chance involved.