Inference *as* prediction

Jane Watson
University of Tasmania
<jane.watson@utas.edu.au>

Inference, or decision making, is seen in curriculum documents as the final step in a statistical investigation (Australian Education Council, 1991). For a formal statistical enquiry this may be associated with sophisticated tests involving probability distributions. For young students without the mathematical background to perform such tests, it is still possible to draw informal inferences based on data of various sorts, for example by comparing two graphical representations (e.g., Watson & Moritz, 1999). In doing so it is important to be able to state the assumptions that are the foundation for the decision made (Whitin, 2006). This article considers a straightforward context where students are asked to make predictions. These predictions are informal inferences that can be based on aspects of the scenario, the students' appreciation of the context, and their cognisance of the data presented.

Making predictions in the face of incomplete information can be hazardous. One of the goals of the statistics curriculum is to assist students in making predictions that have a high probability of turning out to be correct, or at least to assist students in being able to judge what the likelihood of being correct is. In this situation there is usually a risk of being wrong and some children (and adults) are not risk-takers. Many aspects of the mathematics curriculum also reinforce a view against making predictions without certainty. Much of the problem solving carried out in the classroom for example results in one correct answer to a problem. Proof is also put forward as a way of ensuring that conclusions are true. In the world outside the classroom, however, students often are required to make decisions where several alternatives may appear reasonable.

As educators, beyond being statistics or mathematics educators, we want students to become critical thinkers who, when asked to make a prediction, can do so by taking into account all of the perspectives available in the context where the prediction is to be made. We do not necessarily want them to ignore all information besides that which might be supplied by a statistic.

In order to assist in understanding the development that takes place during the years of schooling in students' willingness to make inferences, a small study is presented where 30 students in grades 3, 5, 7 and 9 were asked to make a prediction. Three years later, 22 of them were asked to make the same prediction. The range of responses across ages and over time shed light on what teachers can expect in their classrooms. In this case there is also an indication that the school, a private girls' school in an Australian capital city, may have adapted its curriculum in the intervening years. The author was not involved with the school except in relation to the research carried out.

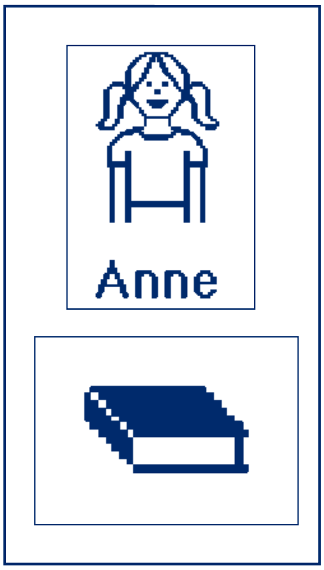

Figure 1. Cards used by students to create a pictograph.

The context of the prediction question was an interview that was part of research into students' understanding of concepts in the chance and data curriculum (Watson & Moritz, 2001). Students were presented with a task to create a pictograph representing the number of books each member of a small group of children had read. To do so, they had small cards that were drawings of named children and single books, as shown in Figure 1. All students could count the books accurately and create a representation, although the form of the presentation varied. Figure 2 shows typical representations as created by the students; not all students worked with exactly the same pictograph. There was an indication in the information presented to the students through the protocol that in total the girls had read more books than the boys (for 2 girls and 2 boys, the totals were 10 and 4 respectively; for 3 girls and 4 boys the totals were 14 and 9 respectively). After the representation phase of the interview was completed, students were introduced to a new student, named Helen, who had come to the class, and asked to predict how many books Helen might have read. Following this another new student, Paul, was introduced and the same question asked. About half of the time Helen was introduced first and half of the time, Paul. Stop now and think how you would respond to this question.

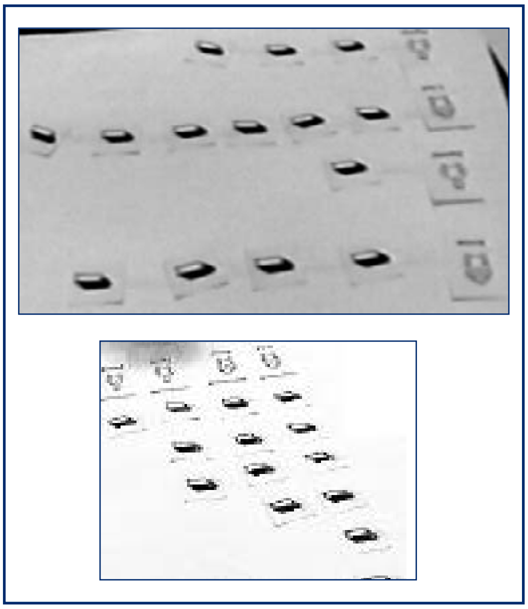

Figure 2. Two typical representations created by students.

Based on the responses, 30 at the beginning and 22 at the end of the three-year period, nine different elements, or components were observed in the responses. Due to the nature of the question, responses could not be judged as "correct" or "incorrect"; there was no right answer. Some elements of responses, however, could be judged as more statistically appropriate than others. The most sophisticated responses might, for example, include several types of components, even contrasting statistical and non-statistical elements.

Examples of each of the elements of responses are presented with the grade level indicated. Grades 6, 8, 10, and 12 responses are from the second interviews. Here all of the responses are from different students. In places the interviewer's questions have been shortened to save space.

Four elements of responses were non-statistical in nature. These usually occurred in isolation. First was the refusal to make any prediction, most often without a reason.

S1: [How many did Paul read?] I couldn't tell because I haven't been given the information. [Grade 5]

In the second case some students made guesses with no justification.

S2: [Paul?] About 3. [What makes you say that?] Nothing much. [Just a guess?] Yes. [Grade 5]

Other responses used of a third element of made-up stories.

S3: [What about if Paul came along. Can you tell how many books Paul might have read?] Umm [pause] Maybe 2. [Why do you say 2?] Because he might not like reading so he might think that reading is a bit boring so that is why he would have read about 2 books. [What do you think about Helen, do you think she would have read a few?] She could have read about 3 because she might like reading, but then she wanted to get this really good series that her friends might have said are really good, but she thought it might have been 10 or 12 books, but then she realised there was only 3 so she just got that set. [Grade 3]

Finally a few students used a fourth strategy of selecting values based on a pattern or a gap in their data displays.

S4: [Paul?] He could have read 2 or 5. [What makes you say that?] Because we are missing numbers all the way along. [Grade 9]

Five elements of responses were statistical in nature. One element was 'reference to the range of values in the representation,' often used in conjunction with other statistical ideas. The following response is an isolated reference to the range.

S5: [Paul?] Between, maybe 1 and 6. [What makes you say that?] Because 1 is the ... [least], or 0, and 6, because he could have read no books or he could have read 6 books, because that is sort of the highest. [Grade 9]

A second element focusing on the mode, referred to as 'most,' was infrequently observed.

S6: [Helen?] I'd say 5 ... because two people read that amount. [Paul?] Five books again. [Because it is the most?] Yes. [Grade 6]

Two elements of responses were associated with centres. Some of the descriptions were based on the 'intuitive idea of middle.' These general references to middle were more likely to be associated with other statistical descriptions, as shown in the next two extracts, than references based on the mean.

S7: [Helen?] About 5. [What makes you say that?] Well the girls seem to enjoy reading the most and basically nearly everybody's read around 5 or 4 except a few people so that indicates that she might like reading as much as they would. [Paul?] Probably about 3 or 4 because all the boys except Andrew don't seem to like reading as much, and so yeah. [How do you decide 3 or 4 particularly?] ... Because Ian only read 1, Danny 2 and Terry 4, with the exception of Andrew who read 6 so most of the boys, the majority read less than the girls and Andrew. [Grade 8]

S8: [Helen?] I say about 5 because it seems to be around the middle of how many books people have borrowed because the smallest is about 1 or 2 and the biggest is about 6 or 7, so it would be about 4 or 5 I'd say. [Paul?] I say he read 3 books because well just from seeing the boys, assume to, like that's the average of how many the boys borrowed. [So you think it might be different for the boys because they seem to have read less?] Mmm. [OK, and how did you determine that it was three?] Um, well I sort of had a look at Andrew and Terry and Danny and Ian and just took a guess of about how many there would be, like how many books Paul had read. [Grade 10]

The other element related to centres was specifically related to the 'mean,' with little interest shown in other features.

S9: [Helen?] Um, about five or something. [OK, what makes you say that ... five?] Um, maybe a bit less, actually four or something. Because whatever ... you add them up and do the average or something like that ... add them together equals 30 and divide them by 7 ... [student uses calculator] Um, 4 point whatever. [Paul?] It would be different, you'd have to add four on so it would be 34 and then divide by 8. [Grade 8]

The fifth statistical element was contained in responses that 'distinguished between the boys and girls.' No student talked about the difference between boys and girls without also mentioning an idea associated with middle. Examples are the responses S7 and S8. A few students included some contextual assumptions alongside their statistically-based predictions.

S10: [Paul?] Well it depends, if he's around the kind of reader that they are and that if he has around the same reading average then we'd add all of these books up and divide them by 4 to find the average. And that would be about how many. [OK, shall we do that?] [counts aloud] 6, 9, 10, 14 divided by 4 is 3.5 [OK so you'd expect him to have read about that many?] Yes, 3 and a half books. [Helen?] Um well if she came along at the same time then no; but if she came along after Paul then it might make a difference. [Right.] Because, um ... Oh no, probably wouldn't because he's just got an average of it and then it would just be divided by the same numbers and so it would still be 3 and a half for Helen. [Grade 7]

In the initial interviews, 25 out of 30 responses were non-statistical in nature, including all of the Grade 3 and Grade 5 responses. Two Grade 7 and 3 Grade 9 responses were statistical in nature. Three years later, 19 out of 22 responses were statistical in nature. The three non-statistical responses were from students now in Grade 6.

It would seem unlikely that the improvement was only due to maturation. The percentages of statistical responses ordered by grade, not year collected, are shown in Table 1. The percentages of Grade 7 and 9 statistical responses in Year 1 are lower than the Grade 6 and 8 percentages in Year 4, even though the students were on average at least a year older at the time of the interviews. It would appear that in the intervening three years between interviews the school focussed on statistical reasoning across the upper primary and middle school years.

Table 1. Percentage of statistical responses ordered by grade.

Grade 3 (Year 1)	Grade 5 (Year 1)	Grade 6 (Year 4)	Grade 7 (Year 1)	Grade 8 (Year 4)	Grade 9 (Year 1)	Grade 10 (Year 4)	Grade 12 (Year 4)
0%	0%	40%	25%	100%	37%	100%	100%

A pictograph representing books read is a relatively simple context within which to ask students to make predictions. It could be used in a classroom to encourage discussion about the criteria to be used in estimation. From a teaching standpoint, how can these responses be used in the classroom to create a culture of including as many components as possible in reaching a sound prediction? Obviously it is important to acknowledge that “Paul and Helen are not here to tell us how many books they read,” so “we cannot be certain of our prediction.” That fact, however, should not stop the class from making the best prediction possible based on all of the information available. It is interesting that some of the interviewed students who knew how to calculate the mean apparently did not see the need to consider any of the other information available, for example, gender. Others who were less precise about “middles” took into account other facets of the data and context. This is encouraging and should be applauded. Another interesting point to discuss would be S10’s prediction that Paul and Helen could have read 3 _ books. The class could discuss the meaning of this in the context. In some contexts 3 _ might have little meaning but here it is possible to suggest reading half a book. What does the class think of this prediction? Also of interest is S10’s presentation of some assumptions about Paul. Other students would likely be able to suggest their own assumptions that could influence the prediction. A class discussion should incorporate as many of these suggestions as possible and it is likely that different classes will arrive at different “best” predictions. Students could be asked to write individual summaries after a class discussion to express their final preferences, again realising that there are no "correct" predictions, just some that are more statistically appropriate than others.

In the classroom it is important to consider the full range of responses, from that of not making any prediction because all the information is not available to that of making thoughtful predictions that recognise statistical characteristics of the data and acknowledge uncertainty. All responses should be handled carefully by the teacher, valuing particularly those responses that contain several elements and promoting a reflective consideration of them. Just telling students to "calculate the mean number of books read" or accepting this response without further discussion does not lead to an appropriate beginning for the appreciation of what statistical inference is about.

Acknowledgement

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Investigation Ideas

- If you have timber of two different lengths, how many different rectangular picture frames can be made? What if you had timber of three different lengths, four different lengths, or n different lengths?
- A calculator has the 6 and 5 key broken. How many different ways can you perform the calculation 65 + 56? What about 65 – 56 or 65 × 56?
- Is it the case that the cube of any number can be written as the sum of odd numbers?