Autumn leaves: Problem posing in an early years classroom

Judith A. Mousley

A neophyte teacher was developing an innovative child-centred teaching style that included half-hour problem posing sessions for mathematics in her Grade 1 in Australia. It demonstrates an alternative approach to early years mathematics teaching and some dilemmas that this raised for the teacher.

THE AUTUMN LEAVES: LESSON ONE

Lunchtime

"Seventy-one on this branch. How many d'you get, John?"

"Ninety-two."

"Ninety-two? Hm, about half way between seventy and ninety."

"Yeah. Eighty? Eighty! Count another one?"

"Yeah."

"Okay, count two more," directed Neil. "John, do that long branch. I'll do this one. Jeremy, go get a jotter and pencil."

So a group of six and seven-year-olds whose football was stuck in a tree had found their problem. From Mondays to Thursdays, their teacher[i] scheduled half-hour sessions for "Problem-solving", always straight after the hour-long lunch break. The children often planned their problems during lunchtime, but these boys had been playing happily before the ball became firmly lodged among the autumn leaves. They'd tried knocking it down with their shoes, then a brick, and they finally rescued it with a yard broom. Now their attention had turned to the number of autumn leaves they had knocked down, compared with the number still on the tree. While they did not know the words "ratio", "fraction" or even "compared with", they shared an understanding of the problem.

Later, the boys confirmed that eighty was a rough average of the number of autumn leaves left on each branch, and had counted fifteen main branches. They had jotted down this information.

"Now the leaves on the ground. Too many to count."

"Could we … could we … do the same thing?"

"Hey? How?"

"Split it. Split it up. Take part of them and … say, "Well, twenty here, twenty in this heap, and there's five heaps."

"Yeah! Let's! Let's make piles."

The boys had dived headlong into the leaves, throwing them at each other and around for a minute or so, then forming their hands like dozer blades and made appropriate noises while they pushed the leaves into about six heaps. Ten minutes later, they were carrying small handfuls of leaves from pile to pile, trying to even them up.

"Twenty-three in this pile. Just count one more pile. There's about twenty. Twenty?"

"Twenty. Five, no six, piles. Twenty, forty, ... hundred, and one more twenty. Hundred and twenty. Hundred and twenty. One hundred and twenty. Write it down."

"There's seven. Ya forgot the seven pile! Get a calculator, John."

As John ran toward the classroom, the bell rang. Lunchtime was over, but the group had its problem.

Problem posing

A few minutes later, Sandi entered her combined Grade 1 (second year of schooling) classroom in an Australian regional city to find four of her pupils huddled over several soiled sheets of jotter paper, a calculator, and the rescued football. Two others were weighing a shoe and a brick. Most of the other children were also getting their "Number problems" organised.

"Well, some of you have started well. Is there anyone without a problem today? There's some rice and containers on the wet table for you."

Seven children without other activities planned moved to the "wet table" table and started playing with filling cups and pouring rice from one container to the other. Some small groups formed to talk about what problems they might explore together using the rice. They collected crayons and paper to record their problems and solutions, as required. On the table, along with the maths equipment, were prepared flashcards for the words "rice," "cups", "containers" and "fill", plus blank cards which children could take to Sandi for help with a particular word. On a nearby wall chart were pictures and words for half, quarter, full, empty, large, small, big, little, and other useful words. Flashcards from the previous few weeks were stored alphabetically in a shoe-holder. A large list of problem starters such as "How many ...?" and "What would happen if ...?" hung on the wall. The groups worked on a variety of problems, some continuing with problems they had started days beforehand.

Everyone seemed occupied before the teacher turned her attention to the obviously excited and busy "autumn leaves" group:

"Robert, tell me about your problem" Sandi urged a boy at the edge of the group.

"Ball. Neil's ball," said Robert.

"What happened to the ball?"

"Tree." Robert pointed upwards.

"Tree? It was in the tree? Neil's ball was stuck in the tree, was it?" [Robert nodded]

"We knocked down the leaves," Tony interjected. "We're trying to find out how many. How many down. How many on the tree. Like, how many of the tree ones are off."

"Mm, I think I see your problem, Robert. Did you help count the leaves on the tree?" [Nodded]

"And on the ground?" [Nodded]

"Where were there more leaves, Robert? "

"On tree."

"Okay, there were more leaves left on the tree. More than on the ground? [Nodded] Now remember, Robert, do some of the drawing and coloring. And you could write the numbers the boys tell you. Okay? Nado, what are you drawing?"

> “The tree. See there were lots of leaves on the tree. This tree here. Now these leaves … here … were … up here. We knocked them off. Our problem is how many — how many of those.”
>
> “So you’re going to find out what part of all the leaves—out of all the ones that were on the tree—are now on the ground? Is that right? Do you want me to help you write your problem? I’d better, I think.

There was general agreement, so after a bit more clarification, the teacher wrote under their drawing, “What part of all the leaves are on the ground?” She left the group to continue solving their problem, with a reminder that they needed to record their solution with writing as well as drawings, and that she would ask any one of them to explain their findings to the class.

As Sandi visited other groups, another two of the footballers’ group told Sandi they were going outside. They had been using a balance to weigh a brick and a shoe, using wooden MAB blocks for weights, and had written down the mass of each. Sandi asked them to talk about their problem. Danny explained that they had noticed that the brick that had been thrown at the football had dug into the soft ground further that the shoe when they had landed. Danny and his friend now wanted to measure “how big the holes are”. Since they had not been taught formal measurement units, Sandi was interested to see what they would use for measuring the holes, but did not probe further.

Sharing time

After half an hour, the children were called to the mat. Each group gave a report on what they had done “for today’s problem work”. Danny and Phil had filled the indentations with sand, using a graduated measuring cup. The brick had made a hole that they had filled with “a bit more than a cup” of rice, and the shoe’s hole needed only “less than half a cup”. The brick had weighed “seven big [MAB] blocks” and the shoe “one block and seventeen flats”. It seemed that heavier objects made bigger holes, but the two boys had discovered more than this. “Like, if it’s double, then it’s probably about double” was the closest Phil was able to get when explaining their revelation. “Tomorrow we will use doubles—real doubles.”

Questioning from Sandi brought out their plans to drop a variety of objects in the sandpit the next day, to see how much sand would be needed to fill holes made by objects of different weights. Sandi, through probing further, understood that they would first select objects whose relative masses were known, so first they would need to find such pairs of objects from the classroom that were (say) twice/half as heavy as each other. She led a class discussion on “families of weights” where “Dad might weigh twice as much as Bill, and Bill weighs twice as much as little Sue”. Several such “families” were drawn on the chalkboard. The children were asked to find a “family” of books (about double the size or mass) from the library shelves in the room, and most seemed to understand the concept well. They were asked to bring along to school the next day any families of objects that they found at home. One child said she has a “family of dolls that fit inside other ones”, and that started quite a bit of chatter about what others could bring.

Some other children expressed an interest in joining Danny and Phil who would be experimenting with dropping objects into sand, and they made useful suggestions about objects that could be dropped—especially some of standard weights. The teacher didn’t

formalise the membership or roles of the group, but moved on to ask a member of the "autumn leaves" group to give a report on their problem.

Without understanding the term "average", the leaves group had decided that there were the equivalent of "fifteen middle-size branches" on the tree in question. Each branch had "about eighty leaves—some more and some not more". They struggled through an explanation of why they needed to use the "middle-size" idea, supporting their claim that there were "about eighty" leaves on each branch even though the branches were of different sizes. Despite the difficulty of these concepts, most of the other children listened intently. The boys reported that they had worked out with a calculator that there were still about 1200 leaves on the tree. They had been arguing about how to say that number, but they had the right numeral written down.

Sandi made flashcards as they talked, with large figures of any number names mentioned. As she laid them on the floor, she said:

> "So you have been working out [Placing the cards down] 15 ... lots of ... 80. [Pointing] Fifteen branches ... eighty leaves on a branch. [Pointing] Fifteen ... lots of ... eighty. And with the calculator that came to over one thousand! [Pointed] One thousand and [Pointed to the 1 and paused] and some children said "Two hundred". One thousand and two hundred. One thousand and two hundred! It's much more than one thousand. One thousand and two hundred. Well done!"

Julie amazed the teacher when she claimed "But we did the same." With some prompting, Julie reported that her group had been trying to find the middle page of the classroom dictionary. The teacher asked if anyone else had "thought about middles". There was quite a bit of noisy discussion before Danny and Phil realised they had, when they had decided to find objects between the shoe and the brick in mass. Alison had, in ordering the size of five rice containers by their capacity. Robert had, said Neil, when he had helped make the piles of leaves "all middle-size" at lunch time. There was excited chatter and laughter about the similarity of their problems. Many of the children started talking about "middles".

The reporting session had taken a more than half an hour and these young children had been thoroughly engaged. I realised that Sandi had give only three gentle reprimands to children whose participation had waned.

Diary writing

As usual, the lesson that followed Problem solving was Diary Writing. Some children chose to copy their problem solutions into their diaries, while others wrote and drew about their recent experiences. Other children continued writing that had been commenced the previous day, or just illustrated other diary pages. Sandi talked with most of them, asking each to read their "stories".

THE AUTUMN LEAVES: LESSON TWO

Interviewed after the lesson, Sandi claimed she was generally happy with the problem-solving lesson but was pondering how to follow it up. She had planned to extend pupils' options for recording findings through making "column graphs" with squares of paper. But perhaps it was more important to drive the concept of averages further or to follow some other leads, such as relative mass, that had arisen through the lesson. Or should she just let the children work further on their problems without trying to direct their learning? Her

decision was that the time was ripe to introduce bar graphs as a way of recording solutions. Besides, she had written “Visual representation—bar graphs” in her program, usually planned about one week ahead, as is traditional in many primary schools.

Bar graphs

Sandi explained to the class that the little squares of paper in front of them (about 30 for each child, in ice-cream containers) “… are like yesterday afternoon’s autumn leaves”. There were lots left on the tree and a few on the ground,” she said. Sandi asked them to make a long row of “leaves” for the ones on the tree and a shorter row for the ones that fell to the ground.

The response surprised her. Three children made a large and a small circle from their squares, several made large and smaller piles of paper squares and two pupils made large and smaller rectangles, borrowing enough squares to fill in any gaps. Two children laid out two crooked lines, focusing on one having double the number of the other. The others, though, followed her instructions and made two lines of paper squares, some working in small groups spontaneously, pooling their “leaves”. As Sandi asked the children to paste their “graphs” onto larger sheets of paper, her dilemma returned. She had intended to introduce the idea of *bar* graphs. “How important do you think it is that bars be straight, at this age?” she asked me.

SANDI’S DILEMMAS

This neophyte teacher had not told her colleagues about her daily problem-solving sessions, even though she joined in their staffroom discussions about teaching. She “knew” that the principal would not approve and that “none of the other teachers would agree with” her ideas. She had been teaching less than a year and was not known as “one of the better young teachers”. Her classes were often noisy (“Chaotic!”, said the principal, “like bees in a bottle!”) and she felt that at least one other teacher thought the children to be “uncontrolled”. Some of the parents, however, had commented on the new enthusiasm their children had for school, and a few of the other young teachers had come to her to talk about how to enliven their own programs.

Sandi felt that she would not yet be able to convince all parents and her colleagues that the children were “covering the syllabus”, so she made a point of teaching traditional number concepts on Fridays and having the children “take home their page of sums”. Sandi jokingly claimed that if the children took home some “sums”, concerned parents could then teach the children all weekend if they wanted to. Fridays were also the only days that the calculators were not freely available. Sandi was aware of inconsistencies in these approaches, but claimed she was trying to get the confidence to allow the set mathematics curriculum to be studied entirely through a problem-posing approach.

However, despite Sandi’s feelings of insecurity about the teaching style she is developing, and despite her inexperience, she did seem to be acting very competently and successfully during these sessions. Over a period of five months during her first year of teaching, Sandi had moved a large group of very young pupils to be interested in seeking out real-word mathematical problems and working together to solve them. She was encouraging active involvement of all children at each stage of problem-posing, recording

the problem, recording solutions, talking about their thinking, and learning about alternative ways of exploring the mathematical concepts. She encouraged individual differences in the way children explored and communicated ideas, but managed to have the children see commonalties across a range of problems and to learn from each other.

In their reporting, the children were encouraged to use oral and written language, drawings and concrete materials. In only their second year of schooling, they calculated, estimated, and generalised confidently. Sandi used the children's own language when discussing concepts and extending their thinking, but introduced more acceptable and powerful terms when she was sure that some common understandings had been reached.

In the apparently chaotic setting of her non-traditional problem-posing sessions, Sandi was encouraging independence and self-control. Children knew they are trusted to work in any area of the school, and she claimed that behavioural problems rarely occur because the students were so involved in their own interesting problems. She did not agree that her classroom is unduly noisy, although she seemed somewhat embarrassed about more experienced teachers' opinions of her teaching. Rather than having a chaotic class, Sandi felt that the children were well organised: they help themselves to equipment and rarely had to be reminded to put it away. They followed the classroom routine happily. But Sandi's students' independence and self-determination seemed to extend a lot further than this: to the very purpose of their schooling. They are taking responsibility for a small part of their own education and for the learning of their peers. The relative lack of direct teaching allows her pupils to take ownership over their learning activity—to turn it from free or directed play to purposeful mathematical investigation.

Sandi was continually surprised by the difficult concepts that the children created and came to reason about in their own ways. She told me that the traditional activities she saw and practised during practicum throughout her training had seemed limiting and "often wasteful of the children's potential". For Sandi, the mathematics curriculum seemed "like an endless list of ideas to be taught in a linear, planned progression". Her curriculum is not orderly, but has taken on the structure of a multi-dimensional web with many concepts related to others. Her pupils are discovering pattern, order, and relationships in numeracy that reach far beyond those expected in the Grade 1 mathematics curriculum.

Sandi's students are fairly free to create their own mathematical units for measurement activities, and their own ways of measuring. Their ingenuity makes her aware of limits to her own imagination as well as that of most curriculum materials. While she feels it is important to eventually teach measuring techniques and standard units, she is perceptive enough not to destroy this creative flair by modelling the "right" methods at this stage. Sandi is delighted at the integration of curriculum areas that the problem-solving sessions are encouraging. Her children pose problems arising from art activities, stories, physical education classes, science investigations, and social studies topics as well as objects in the classroom and schoolyard. They construct problems with their parents, siblings and peers; as well as working on problems arising in the classroom, playground or home. Sandi claims, laughingly, that their interest "knows no boundaries".

Sandi spends a lot of time pondering over these pedagogical dilemmas and discussing them with me. She believes strongly that children need to construct mathematical concepts, rather than have them "passed on" through a series of pre-planned activities and direct teaching. "The children are learning <u>how</u> to learn and use

maths. They understand a wide range of mathematical concepts intuitively". Also, Sandi had a strong belief that these sessions cater for every child—from Neil, the "born organiser", to Robert with his mild learning disability. What worries her is not whether this is an effective way to teach, but how often to use it, "how much to trust it", and how to improve her own input into these sessions. Only after she has found some solutions to these questions will Sandi feel able to defend her intuitive teaching style with confidence—"… and evidence" she added.

CONCLUSION

This tension between a traditional linear curriculum and the realisation that it is important for children to be supported in a more individualised, child-centred quests for knowledge arises frequently in discussions about mathematics education. There is still a powerful expectation of parents, administrators, and most teachers that a procedural syllabus will be "covered" — and this is increasing with the use of broad-based state and national testing.

Mathematical "knowledge" to be mastered exists, for most people, outside of our understandings of the mathematics in our environment. A socially constructed body of knowledge and skills has been objectified through pedagogical traditions and text to the extent where many of us have ceased to realise that the question "What is mathematics in early childhood?" can be asked. As Sandi noted, "The term *real-world problems* is laughable when you read many of word problems in textbooks. Adults never do many of those problems, let alone children".

I am not advocating that established curricula be scrapped. That would be folly as we move to the implementation of the new Australian national curriculum. The point I wish to make is that any curriculum could be used in the evaluation of what *has been learned* more than a prescriptive record of what *is to be learned*. Nor do I contend that all teachers should follow only the interests of their students in setting up maths activities. However, early years school teachers could be encouraged more strongly to expand their perceptions of mathematics as well as to experiment with the notion of child-centred leaning. Most importantly, administrators and teaching colleagues could provide more support for the "Sandis" within our institutions. We could all appreciate more those teachers who are willing to experiment with and develop alternative pedagogical beliefs and practices as well as those willing seek and sustain, rather than control, the development of individualism in early years classrooms.