Max's statements

http://topdrawer.aamt.edu.au/Reasoning/Big-ideas/Deduction/Deductive-reasoning

A year 7 teacher described the following scenario to her mathematics class.

> Playing around with adding integers (whole numbers), Max came to two conclusions.
>
> - The sum of any four even numbers is a multiple of 4.
> - The sum of any three consecutive numbers must be a multiple of 3.

The students worked in small groups to try to work out if both of Max's statements were true all of the time.

Playing around with four even numbers, they found that it was quite easy to choose examples that worked (e.g. 2 + 4 + 6 + 12 or 12 + 16 + 20 + 40). But the teacher challenged them to prove that the statements were true for *all cases* or to disprove the statements.

After some deductive reasoning, many groups concluded that only one of the statements was true for all possible sets of numbers, while the other statement was true only some of the time. Some groups used both proof and counter examples.

Ann, for example, reported that her group had deliberately tried to create an example that did not work (a counter example).

> Well, I knew that the sum of four even numbers is always even. But not all even numbers are multiples of 4 – 14 for example. So I wanted to find some multiples of 4 that add to an even number that is not a multiple of 4. So I worked backwards from 14. I made 14 from even numbers: 2 + 2 + 4 + 6. Bingo! Then I knew that the sum of any four even numbers does not have to be a multiple of 4.

This is called 'disproof by counter example'.

About the second statement, Ann said:

> I was thinking that in any three consecutive numbers, one of them must be a multiple of 3. If the first one is a multiple of 3, then the second number is one more than that multiple of 3 and the third number is two more. 1 and 2 is another 3. So therefore we have some identical multiples of 3, plus 1 plus 2. Then I thought about when the multiple of 3 is the second number and then the third number, and that works the same. So I realised … I concluded that … that the sum of the three consecutive numbers must be a multiple of 3.

The reasoning involved here is exactly that involved in more formal mathematical proof, in that it is a *reasoned argument building on accepted truths*. The reasoning is a chain of deductive steps leading up to a conclusion.