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Title of chapter/article	Developing Fraction Sense Using Digital Learning Objects
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Copyright owner	The Australian Association of Mathematics Teachers (AAMT) Inc.
Published in	Fractions: Teaching for Understanding
Year of publication	2011
Page range	153–166
ISBN/ISSN	978-1-875900-68-8

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CHAPTER 15.

DEVELOPING FRACTION SENSE USING DIGITAL LEARNING OBJECTS

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A strong "sense of fractions" is fundamental to developing conceptual understanding of rational numbers, and includes flexibility with visual representations of fractions, a sense of the quantities represented by fractions and reasoning about relationships between the numbers. Research has shown that it is important to represent mathematical ideas in multiple ways, including real contexts, physical models (manipulatives), pictures, verbalisations and written symbols (Lesh, Cramer, Doerr, Post, & Zawojewski, 2003). For fractions, this means children need to explore a range of contexts, representations and task types such as drawing shapes, cutting up objects, sharing sets of items, folding paper strips and string, naming fractions and locating fractions on number lines. Understanding mathematical constructs is also dependent upon the ability to connect and transfer meaning between different representations of the same ideas (Goldin & Shteingold, 2001; Kamii, Kirkland, & Lewis, 2001; Lamon, 2001). Manipulating physical representations of fractions can support the development of strategies for processes such as comparing, finding equivalence and performing operations, but such activities also need to be firmly connected to symbolic representations (written fractions).

The use of interactive digital resources, called learning objects (similar to virtual manipulatives or applets) has also been shown to support the development of the conceptual understanding of fractions (Gronn, Clarke, & Lewis, 2006; Suh, Moyer, & Heo, 2005). A major advantage that digital resources have over other physical models is that different representations can be dynamically linked, allowing experimentation with cause-and-effect relationships that make explicit the connections between the different representations. For example, the learning object illustrated in Figure 15.1 automatically responds to changes made to the diagram with corresponding changes to the symbolic form of the fraction, and its position on the number line. The user is able to explore cause-and-effect relationships through the immediate feedback provided by the learning object.

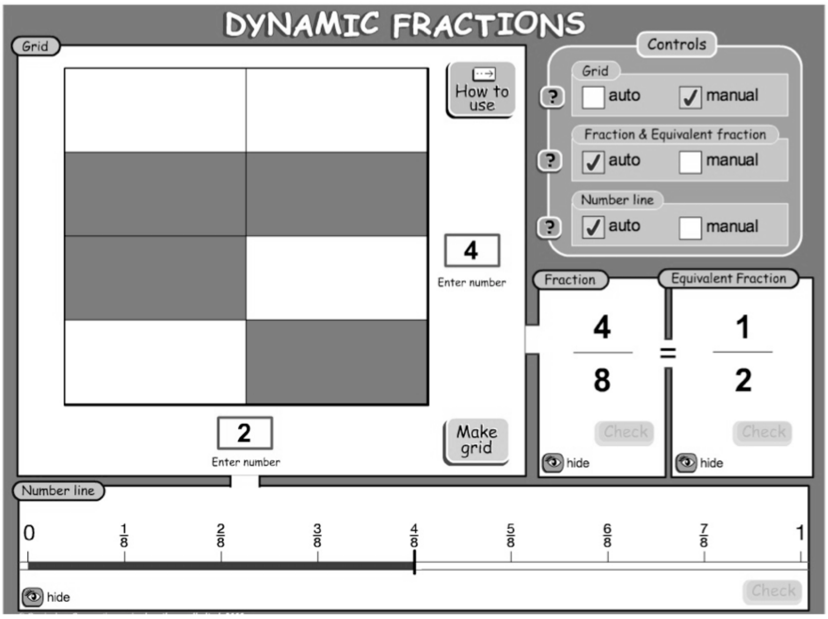


Figure 15.1. Screen shot of a learning object that dynamically links three representations. (*Dynamic Fractions*, Education Services Australia Ltd, 2003. The Learning Federation digital curriculum resource L134).

The purpose of this chapter is to describe ways in which digital resources can be used to help develop fraction sense. Although specific learning objects have been selected they are intended to be indicative of learning opportunities afforded by a range of similar digital resources, and so the teaching/learning suggestions could be adapted to fit the available resources.

Components of fraction sense

Fraction sense is not a clearly defined term but, in research and educational literature, it seems to be generally used to refer to a collection of fundamental perceptions and conceptions about fractions rather than a single mathematical idea, and different educators might itemise this collection of notions in different ways. However, fraction sense is consistently associated with building *conceptual understanding* of fractions and as a basis for developing appropriate strategies in mathematical tasks, rather than associated with procedural understanding. Implicit in this view of fraction sense is that components of fraction sense can accumulate over time through experience and familiarity, rather than through explicit teaching of procedures or rules. It follows that learning experiences that allow exploration and experimentation with fractions might support the development of fractions sense. The following list is an attempt to summarise some of the fundamental ideas about fractions that could be considered as components of fraction sense.

Fraction sense involves:

1. Understanding that, particularly in the part-whole model, fractions refer to *equal* parts of a whole and being able to name fractions both verbally and symbolically (written).

   Teaching tip: Although children need to understand the information communicated by the numerator and denominator in describing a part-whole relationship, care needs to be taken when "reading" fractions. If the numerator and denominator are read as two separate numbers, such as "two out of 3" or "two over 3", this creates a barrier to perceiving the fraction as a number itself. It is therefore better to say "two-thirds".
2. Understanding the relationship between the number of parts and the relative size of the parts. This involves realising that the more parts a quantity is divided into, the smaller the pieces become. When connected to written fractions this means, "the larger the denominator, the smaller the parts".

   Teaching tip: Students might use this understanding as a strategy for comparing unit fractions (for example, to decide that $\frac{1}{4}$ is smaller than $\frac{1}{3}$) but may need prompting to realise this strategy is not so helpful when comparing non-unit fractions (for example, $\frac{3}{4}$ and $\frac{2}{3}$).
3. Having a sense of the size of fractions in relation to one whole, including whether they are less than one (e.g., $\frac{1}{3}$), equal to one (e.g., $\frac{5}{5}$) or greater than one (e.g., $\frac{5}{4}$).

   Teaching tip: This often supports strategies around using benchmarks for estimation, particularly when ordering fractions. For example, the order of the fractions $\frac{2}{4}$, $\frac{5}{6}$ and $\frac{1}{3}$ can be determined simply by deciding if each fraction is less than, greater than, or equal to one-half. However, this strategy will not be sufficient in all cases, so students need a range of strategies to draw on.
4. Being able to instantly recognise and name representations of commonly used fractions (e.g., halves, thirds, quarters and fifths) when only that part is visible, as in the task of identifying the fraction represented in Figure 15.2. In this type of task, the "whole" has to be inferred from the "known" part and visualised. It becomes more

challenging when non-unit fractions (e.g., $\frac{3}{4}$) are represented, as in Figure 15.3.

Teaching tip: Consider the strategies that can be used to quickly recognise a fraction. What fraction do you think Figure 15.2 represents? How do you know? Did you just recognise the fraction through familiarity with the shape? Did you visualise the rest of the circle? Did you imagine how many of the pieces would be needed to complete the circle? Did you use the same strategy for Figure 15.3? Awareness of a range of strategies will help some students develop stronger fraction sense.

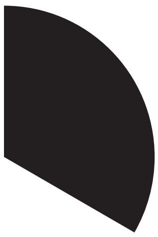


Figure 15.2. What fraction of a circle is represented?

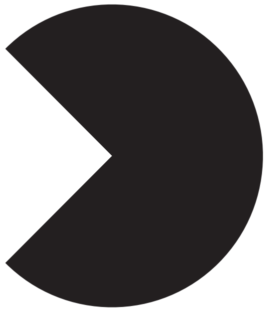


Figure 15.3. What fraction is represented by this part of a circle?

5. Being able to visualise, estimate and create representations of fractions by partitioning "wholes", using a variety of models (e.g., areas of various shapes, strips of paper or string, groups of objects). It is important to avoid always presenting students with stereotyped pre-prepared representations of fractions as this develops automatic responses rather than thinking and visualising.

   Teaching tip: Children have difficulty creating their own representations of "odd" numerator fractions (like thirds or fifths) because they cannot begin with halving (as in quarters or eighths), so they need lots of practice in folding paper strips, drawing shapes and using digital resources to build mental images that will allow them to visualise and estimate these fractions.

Children who have developed these basic components of fraction sense will then have the capacity to apply their mental images of fractions and conceptual understanding to build more complex concepts and processes. For example, being able to imagine the same square divided into halves, quarters and eighths will allow a child to comprehend a representation that contains several different fractions, as in Figure 15.4, and to see how $\frac{1}{2} + \frac{1}{4} + \frac{2}{8}$ must equal 1 whole.

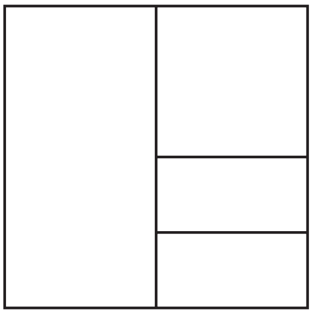


Figure 15.4. Diagram representing $\frac{1}{2} + \frac{1}{4} + \frac{2}{8} = 1$.

Applying fraction sense enables consideration of relationships and comparisons between fractions, which can be demonstrated through forming logical estimations and explanations in problem solving. For example, using fraction sense, the following problems can be solved and the solutions fully justified without making any calculations or physical representations:

- Which is larger: $\frac{1}{3}$ or $\frac{1}{5}$? (Note that to sensibly answer this it is necessary to decide whether both fractions refer to the same size whole, in the sense that $\frac{1}{5}$ of a family-pizza may well be a larger quantity than $\frac{1}{3}$ of a mini-pizza)
- Would $\frac{2}{3} + \frac{1}{4}$ be less than one or greater that one?

Using digital resources

The next section presents ideas for using three learning objects with primary students to support the development of fraction sense. These three learning objects, designed by The Learning Federation (Australian and New Zealand government project) are *available to all schools* (and universities) in these two countries, usually through each state's teacher resource website[2]. For the benefit of other readers, a related digital resource from the open-access NLVM website[3] has also been noted.

The digital resources can be used on a large screen or interactive whiteboard (IWB) for whole class teaching or group-work, or can be used for paired or individual work on computers or handheld devices. When using a learning object as a display for a large group or the whole class it is important to plan teaching strategies that engage all the students with the mathematical content and encourage student-to-student interaction, and avoid the superficial engagement with the lesson content that research has shown to be common when using interactive whiteboards (Hall & Higgins, 2005; Zevenbergen & Lerman, 2007).

[2] See "Access information" at *The Learning Federation* website (www.thelearningfederation.edu.au).
[3] National Library of Virtual Manipulatives (http://nlvm.usu.edu/)

Combining interaction with digital and non-digital representations of fractions can provide rich learning experiences that cater for a range of learning needs, so the task examples presented in the following section incorporate both types of resources. The different task-forms also offer flexibility in lesson organisation, particularly when access to hardware is restricted. Some students can be working with computers while others use physical resources, like paper models or modelling dough, to solve similar problems. However, the digital resources also offer particular design features, known as *affordances*, for enhancing learning, that are not provided by the non-digital resources. These affordances are highlighted in the teaching suggestions to emphasise the advantages of using the learning objects.

While a range of learning objectives might be associated with the four selected learning objects, they have been chosen for their potential to develop skills in the creation of representations of fractions through partitioning, visualisation of fractions and comparison of fractions.

Creating representations

Description of learning object

The Shape Fractions learning object is an open-ended tool that allows selection from six shapes that can be partitioned into parts using dividing lines. When equal parts have been formed, these parts can be selected (shaded) and the fraction's label can be displayed, as can any equivalent fractions that are formed (see Figure 15.5).

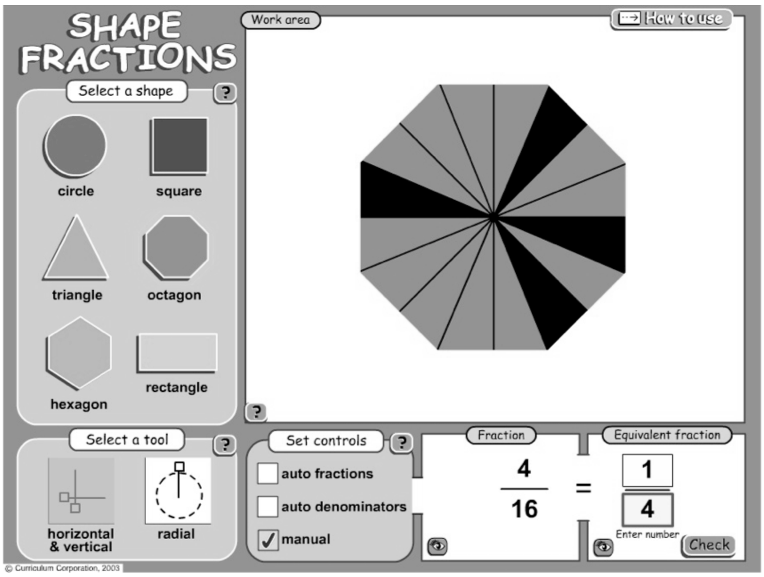


Figure 15.5. Screen shot of Shape Fractions learning object.
(Shape Fractions, Education Services Australia Ltd, 2003.
The Learning Federation digital curriculum resource L135.).

Affordances of the learning object's design

- Selection of six different shapes.
- Radial dividers and vertical/horizontal dividers.
- Direct and dynamic connection to the symbolic form of the fraction, so the denominator changes with the number of equal parts created, and the numerator changes with the number of parts selected. For example, Figure 15.5 shows a hexagon divided into three equal parts, with two of the parts selected (black) and the resulting fraction symbol of $\frac{2}{3}$.
- Display controls provide the option of automatically displaying the written fraction or setting it to "manual" so the numbers have to be entered by the user. Also, any equivalent fractions can be displayed or hidden.
- Feedback on correct and incorrect actions is provided to allow self-correction.

Digital tasks

For tasks 1 to 3, set the controls set to "Manual" and "Equivalent Fraction" switched off, to enable either the teacher or the students to decide the fractions that will be created. These tasks work well with pairs of students.

Task 1

Individuals or pairs use as many different shapes as they can to represent a specified fraction, for example, $\frac{2}{3}$. It might be useful to prepare a recording sheet with the six shapes on it to enable quick record keeping.

Task 2

Each pair of students can choose one shape and represent as many different fractions as they can with that shape. Labelled sketches could be recorded to facilitate comparisons, groupings and discussion.

Task 3

One student nominates a fraction and the other finds a way to represent it. The teacher should encourage discussion of the students' strategies for choosing suitable shapes and types of dividers (radial or vertical/horizontal), as well as sharing of observations made about the fractions created (e.g. one-third can look different depending on the shape that was divided, or the relationship between the number of dividers used and the number of parts created). Stimulus questions could include: Are some shapes better suited to some fractions? Which fractions can be represented using more than one shape? Why cannot both types of dividers be used with every shape?

Task 4

Students explore ways to represent the fractions suggested by the learning object. For task 4, set the controls to "auto fractions" or "auto denominator" and switch off "Equivalent Fraction". Working in pairs will encourage discussion and experimentation. The fractions generated by the software will ensure the students explore a variety of fractions.

Task 5

This task involves direct teaching with the whole class, using a projector and screen or an interactive whiteboard. Set the controls to "Manual" and switch off "Equivalent Fraction". The teacher specifies a fraction, such as one-third, and individuals or small groups sketch it on paper using a shape of their choice. Several students come up and create their representations on the IWB to demonstrate how a variety of shapes can be used. Encourage questioning and discussion about the quality of the solutions.

Note: The NLVM learning resources called *Fraction-Parts of a Whole*, *Fraction Naming* and *Fraction-Visualizing* could form the basis of some similar activities as they connect area diagrams of squares and circles to symbolic notation.

Non-digital tasks

Similar activities to the digital tasks can be completed using paper shapes that can be selected, folded and labelled. Students should compare, discuss, sort into groups and display their folded shapes.

Visualising fractions

Description of learning object

The *Matching Cake Fractions* learning object presents tasks that require students to predict the written fraction that best describes the part/s of a circle presented. The students then reconstruct both the symbolic fraction and the area model of the "cake" to check the prediction. Improper fractions are included in some of the tasks (see Figure 15.6).

Affordances of the learning object's design

In the "checking" step of each task (shown in Figure 15.6), the student is able to experiment with the cause-and-effect connection between the magnitude of denominator or numerator and the portion of cake displayed, as these are dynamically linked.

The tasks become more challenging as the student successfully completes each one, with the visualisation and estimation becoming more challenging; for example, three-quarters of a cake that has not been visibly divided into quarters is presented (similar to Figure 15.3). A record of the solutions is automatically generated and can be printed out. Immediate feedback is provided on errors, that coaches the student to self-correct.

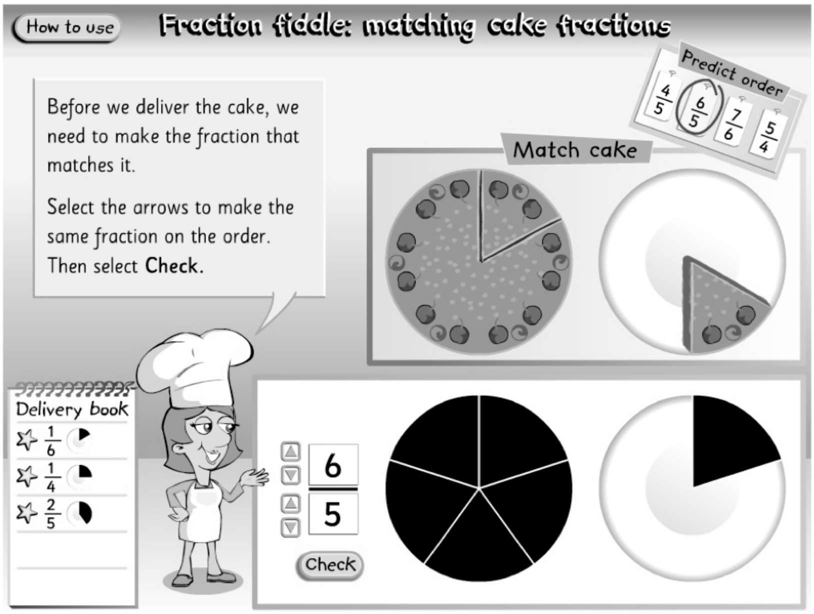


Figure 15.6. Screen shot of Matching Cake Fractions learning object (Fraction fiddle: matching cake fractions, Education Services Australia Ltd, 2009. The Learning Federation digital curriculum resource L2801).

Digital tasks

Task 1

Using a projector and screen or an interactive whiteboard, the learning object is displayed to the whole class. The teacher guides the class through completion of the activities and facilitates discussion of the mathematical ideas and strategies encountered, such as improper fractions and visualising missing divisions. To increase task engagement and mathematical thinking, the class could be divided into teams who agree on predictions, give explanations of strategies and take a turn of carrying out the checking process.

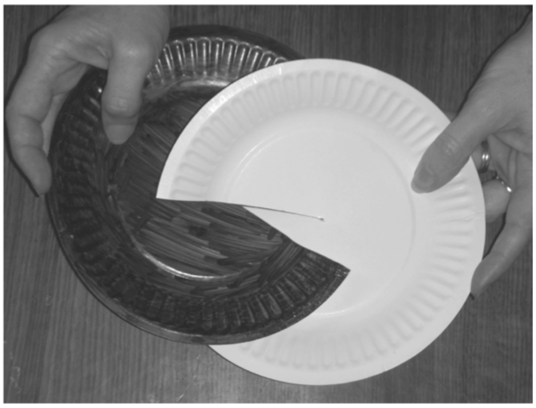
Task 2

Working in pairs, students take turns to complete each of the tasks presented by the learning object. Using the printed record as a stimulus, a debriefing discussion could focus on visualisation and estimation strategies, and on discussing what happens to the diagram when the denominator or numerator is changed.

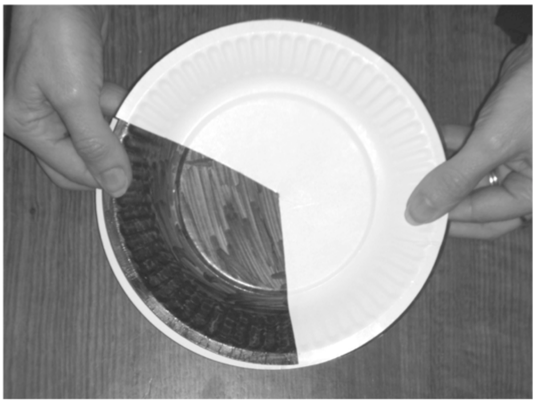
Note: The NLVM learning object called Fractions-Pieces could be used a basis for some similar activities for fractions less than one.

Non-digital tasks

For tasks 3 to 5 each student will need to have made a *fraction-wheel* from paper plates or circles of paper/card (see Figure 15.7).



Make a cut from the edge to the centre of each circle and slide together.



Turn the circles to make fractions.

Figure 15.7. Making a fraction-wheel to model fractions.

Task 3

Working with the whole class, the teacher specifies a fraction and each student estimates that fraction on his/her fraction wheel and holds it up. A few different models can be selected for discussion until the class agrees on the representation that is closest to the specified fraction and how this can be judged. Improper fractions can also be represented by students grouping together to display the required number of wholes and parts.

Task 4

The teacher makes a fraction on a fraction-wheel and flashes it quickly to the class. Pairs or small groups recreate the fraction on their own wheels and decide how to name it. Compare the different solutions and consider why different judgements might be made. Highlight any equivalent fractions that emerge. As a class, discuss how to decide what the denominator should be and what the numerator should be.

Task 5

Pairs of students can set each other fractions to create on the wheels, then record a sketch labelled with symbolic notation.

Comparing fractions

Description of learning object

The *Comparing Non-unit Fractions* learning object presents a pair of written fractions (e.g., $\frac{2}{3}$ and $\frac{3}{4}$) and asks for a prediction of which is smaller (or larger) than the other. Students construct a representation of each fraction on bars (linear model) to compare them, and then complete an answer statement (see Figure 15.8).

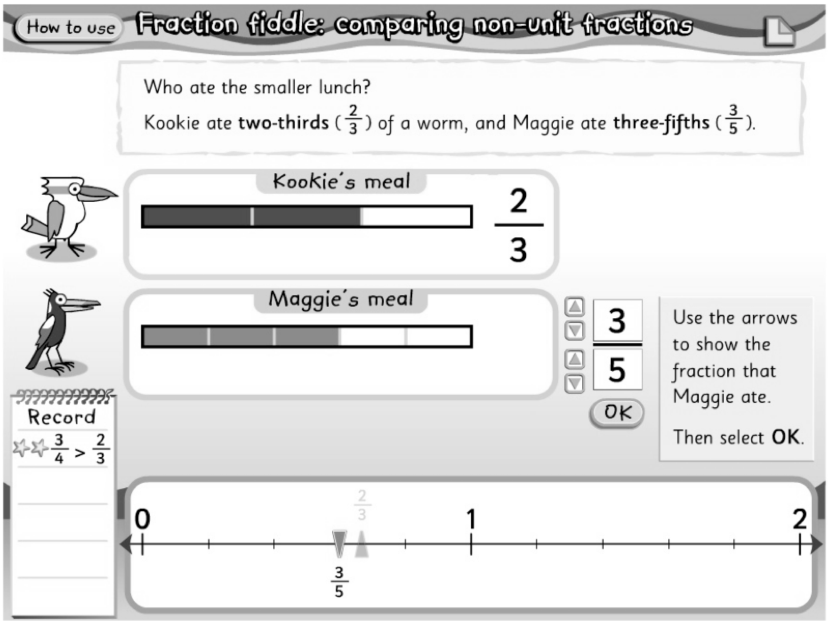


Figure 15.8. Screen shot of Comparing non-unit fractions learning object.
(Fraction fiddle: comparing non-unit fractions, Education Services Australia Ltd, 2009.
The Learning Federation digital curriculum resource L2803).

Affordances of the learning object's design

- The visual model is created by the student manipulating the magnitude of the numerator and the denominator, and so the cause-and-effect relationship can be explored.
- Accurate divisions are made, making comparison of similar size fractions possible.
- The fraction bars are mapped onto a number line so the numerator and denominator can also be seen as combining the form a number.
- A record of the solutions is automatically generated and can be printed out.
- Immediate feedback is provided on errors, that coaches the student to self-correct.

Digital tasks

Task 1

Using a projector and screen or an interactive whiteboard, the learning object is displayed to the whole class. Divide the class into teams, perhaps associating teams with the characters in the learning object, Kookie and Maggie. (If a point scoring method is devised then a game format can be used to increase engagement with the task). Ask students to predict an answer to the problem presented, such as stating which bird has the smaller lunch, and explain the thinking behind the choice. Also ask for prediction of each fraction's location on the number line. Students can then take turns in creating the models to make the comparison and check the predictions.

Task 2

Pairs of students work through the tasks, taking turns at predicting and creating the fraction bars to check. Teacher questioning could focus on the students' understanding of the relationships between the symbolic notation, the fraction bars and the number line. In a follow-up class discussion, students could select a pair of fractions and explain how they know one is larger than the other, and what strategies might be useful when the learning object is not available.
Note: The NLVM learning object called *Fractions-Comparing* also compares fractions and locates them on a number line but uses the procedure of finding common denominators using area diagrams rather than making a direct comparison.

Non-digital tasks

Task 3

Pairs of students predict the larger/smaller of pairs of unit fractions then check by folding strips of paper to represent each fraction. For example, which fraction of the same ribbon is longer, $\frac{2}{3}$ or $\frac{3}{4}$? As a class discuss strategies and reasoning for making predictions and for folding the paper strips accurately.

Task 4

The teacher displays a peg-line to the class with 0 and 1 labelled at each end. Students are asked to estimate the location of various non-unit fractions and peg up label cards. Other students can be asked to devise a way to check or justify each positioning, and explain which fraction is larger/smaller than the other. As a class, discuss strategies and reasoning for locating the fractions and for comparing the sizes of fractions.

Conclusion

The development of fraction sense is fundamental to building conceptual understanding of fractions and the associated mathematical principles. Children who do not have a strong fraction sense are more likely to depend upon taught procedures for working with fractions, and more susceptible to faulty procedures, misconceptions, and flawed reasoning in problem solving. Digital resources such as the learning objects presented here, provide opportunities for students to explore cause-and-effect relationships between representations of fractions and so have the potential to make a strong contribution to the development of fraction sense.

Five general principles for utilising digital resources such as learning objects in the classroom are to:

1. Provide scaffolding for learning through teaching strategies such as introducing the tasks, making connections with previous experiences, questioning during tasks, setting challenges, facilitating discussion and debriefing;
2. Use learning objects in conjunction with other forms of resources and task types;
3. Promote strong interaction between the learning objects and the students, and amongst the students themselves. Be wary of only using whole class approaches such as displaying on an interactive whiteboard, as this often dampens both engagement with thinking and verbalisation of ideas;
4. Consider the main affordances (learning features) offered by the learning object and take full advantage of them. For example, if cause-and effects are observable, encourage the students to notice and talk about them; or if a printable activity record is available, use it for a lesson follow-up or for assessment; and,
5. Plan for utilising the learning objects more than once to allow for building strategies and further experimentation. The Learning Federation resources tend to be designed in series, so look for similar learning objects that provide more/less challenge or emphasise different skills.

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