Title of chapter/article	A primary classroom enquiry: Estimating the height of a tree
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Copyright owner	The Australian Association of Mathematics Teachers (AAMT) Inc.
Published in	Australian Primary Mathematics Classroom vol. 16 no. 2
Year of publication	2011
Page range	3–11
ISBN/ISSN	1326-0286

A Primary Classroom Inquiry: *Estimating the Height of a Tree*

NATALIE BROWN, JANE WATSON, SUZIE WRIGHT and **JANE SKALICKY** take children outside to explore different ways of estimating the height of a tree. WARNING: Do not try the method in Figure 1 in front of your students!

From the beginning of the development of a national curriculum for Australia, numeracy has been a feature of guiding statements. In *The Shape of the Australian Curriculum* (National Curriculum Board, 2009), the foundation for numeracy is seen to be built primarily in the mathematics curriculum but is also reinforced in other learning areas (p. 10). Measurement is one of the key areas of study in mathematics and features prominently in the *Australian Curriculum: Mathematics* (ACARA, 2010). In this set of investigations requiring students to estimate indirectly the height of a tree they are encouraged to use the "power of mathematical reasoning" and "apply their mathematical understanding creatively and efficiently" (ACARA, 2010, p. 1). Specifically, the various measurement techniques involved in the investigations encourage students to use ratio, proportional reasoning and properties of triangles to calculate something that they are unable to measure directly. In the *Australian Curriculum: Mathematics*, development of proportional reasoning is encouraged from Year 3 onwards, and ratio is specifically mentioned in Year 7 (ACARA, 2010, p. 35). Making connections between different types of triangles and using their properties becomes a major focus in the Measurement and Geometry strand from Year 2 onwards (p. 19).

The activities suggested in this article are intended for use with upper primary school students, taking into account that teachers will consider the background and level of the students they teach to ensure they are sufficiently skilled to carry out the investigations successfully. The investigations presented here were used by the authors during a half-day professional learning session with middle school teachers from five rural schools in southern Tasmania (as a part of the ARC-funded research project “Mathematics in an Australian Reform-Based Learning Environment” (MARBLE)). They have also been used with pre-service primary and middle school teachers as part of the Bachelor of Teaching program in the Faculty of Education at the University of Tasmania.

Although a problem that is set to measure the height of a tree in a text book may seem purely academic, with minimal real-life application, the methods used in these investigations are in fact employed in fields such as forestry where knowing the height of trees is necessary for safe and efficient felling and logging. Architects, planners and surveyors also use the same principles to measure the height of buildings, land formations, etc., often using informal measurement techniques, such as the method described in Investigation 2, to make reasonable estimations. Estimation is encouraged in the *Australian Curriculum: Mathematics* from Year 5 to “check the reasonableness of answers” (ACARA, 2010, p. 27). Trees are used in these investigations as one would expect to see trees in and around most school yards. In highly congested, urban environments, however, or in dry, scrubby bush areas where tall trees are less readily accessible, the investigations can be modified so that students measure the height of buildings or other tall structures. For example, Cavanagh (2008) encouraged students to use ratio and the principle of similar triangles to measure the height of the school flagpole. Regardless of the structure being measured, the hands-on approach to learning used in the three investigations allows students to explore seemingly abstract, disconnected concepts in a meaningful, enjoyable way.

Investigation 1

Framing the activity

Using the Native American Indians’ method, estimate the height of a tree.

As the *Australian Curriculum: Mathematics* states, “Mathematical ideas have evolved across all cultures over thousands of years” (ACARA, 2010, p. 1) and this investigation explores the interesting and unusual method used by Native American Indians to estimate the height of a tree: they would bend over and look through their legs. Asking students to try this method as an introduction to the more formal investigations can encourage interest in the topic and stimulate student learning. Students can use their estimates from this activity to make comparisons with those collected in the subsequent investigations in order to help determine if the method used by Native American Indians does in fact work.

Collecting data

For this activity, students work in pairs. Starting at the base of the tree, Student A

Figure 1. The method used by Native American Indians to measure the height of a tree.

walks away from the tree, stopping at regular intervals. At each stop, the student bends over and looks through his/her legs at the tree (see Figure 1). Student A continues to do this until reaching a point where he/she is just able to see the top of the tree from the upside down position; that is, the entire tree can be seen between his/her legs. Using a measuring tape or trundle wheel, Student B then measures the distance from the base of the tree to Student A. This distance is approximately the height of the tree. The students swap roles and record the distance (height of the tree) each time.

Data representation

Students can use a table to record their estimations.

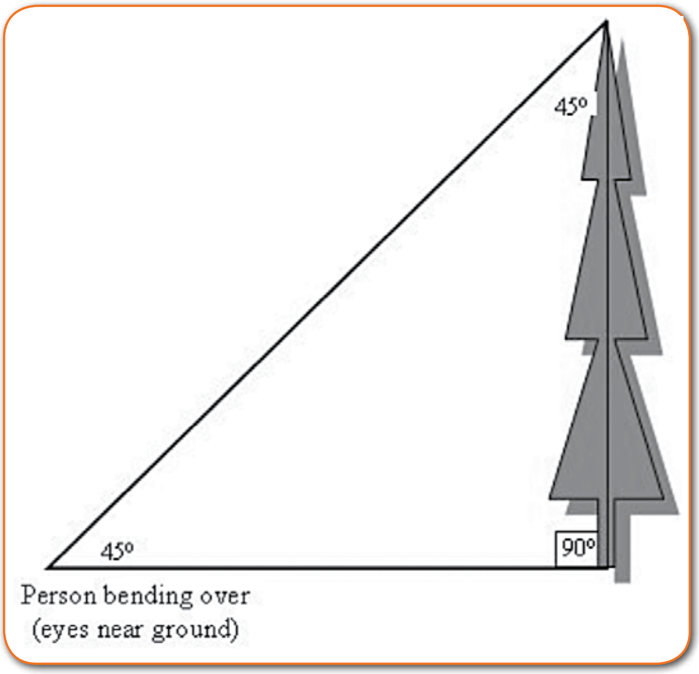

Figure 2. A pictorial representation of the isosceles right triangle formed when looking at the tree from between the legs.

Student	Estimated Height of Tree	Student	Estimated Height of Tree
Millie	4.7 m	Jack	4.4 m

Thinking about the mathematics

The reason that this method works is that for normal, fit and healthy adults (who can bend over in such a way), the angle that is formed as they look through their legs is approximately 45 degrees. Using knowledge of angles and sides of a triangle, this means that the angle between the tree trunk and the ground is reasonably close to 90 degrees. This leaves 45 degrees as the top of the triangle, forming an isosceles right triangle with two equal sides (Figure 2). Hence, the height of the tree and the distance from the tree to the person are about equal and knowing the distance to the tree gives one a good idea about the height of the tree.

Investigation 2

Framing the activity

Using informal measurement techniques, estimate the height of a tree.

When architects want to estimate the height of a building they use a small object to eye off the number of doors to the top of the building.

Dr Steve Watson
Sustainable Building Consultant

Figure 3. Using a pencil to estimate the height of a tree.

Collecting data

For this investigation students need to work in pairs. Each pair needs a pencil and measuring tape or trundle wheel.

- One student stands at the base of the tree.
- The other student slowly moves away from the tree, holding a very small pencil (or using his/her hand), until the length of the pencil matches the height of the other student (refer to Figure 3). The base of the pencil will be at the foot of the student and the top will be level with the top of the student's head.
- Now from this estimation, count how many pencils would be needed to reach the top of the tree. This can be done by carefully placing the pencil in the view of the tree, and counting how many times the pencil can be stacked to reach the top.
- With the measuring tape or trundle wheel, measure the height of the student standing at the base of the tree and multiply this height by the number of pencils counted earlier.
- Swap jobs and repeat the process using the same tree.

Data representation

Students can use a table to record their measurements.

Students can create a graph of their estimates and discuss reasons for why they are not the same. Reasons might include: inaccuracy in measuring the height of the student; individual experience of, and accuracy in, estimation; inaccuracy in using the formula to calculate the height of the tree; and the direction in which the student walked away from the tree and the impact of this on the visual location of the "true" top of the tree.

Student	Estimated Number of Pencils	Height of Student at Base of Tree	Estimated Height of Tree *(estimated number of pencils × height of student)*
Jesse	3	150 cm	3×150 cm = 450 cm or 4.5 m

Thinking about the mathematics

How does this method of estimating the height of a tree work? Students should be encouraged to think about the principles of proportionality. In this investigation, the students position themselves so that the height of the pencil is proportional to the height of the student standing at the base of the tree. Therefore, multiplying the estimated number of pencils by the height of the student will give the estimated height of the tree; the height of the student is a divisor of the height of the tree.

Figure 4 offers a pictorial representation of how this method works. Imagine, for example, that the person in Figure 4 is 150 centimetres tall, and it appears approximately four pencil lengths are needed to reach the top of the tree; then the approximate height of the tree is 4 ×150 cm = 600 cm, or 6 metres.

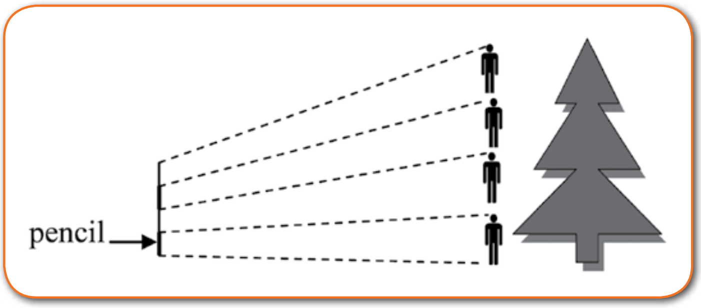

Figure 4. A pictorial representation of using the principles of proportionality to estimate the height of a tree.

This investigation encourages students to estimate by using the "chunking" method. As seen in the example in Figure 4, the tree being measured is broken into manageable chunks that can be added together to estimate the total height. This method is used as it is often easier to estimate or directly measure the shorter chunks than to estimate the whole length as one (Van de Walle, 2004, p. 334). Van de Walle describes a similar estimation strategy of using a familiar benchmark, such as a metre stick or ruler that can be mentally compared to an object (p. 334). For example, one can estimate that the tree is about as tall as eight metre sticks, or using Steve Watson's example, the height of the building is four times the height of the door.

Investigation 3

Framing the activity

Using knowledge of ratio, estimate the height of a tree.

Collecting data

One of the simplest ways to estimate a tree's height requires a sunny day. For this investigation, students can work independently or in pairs, and need a long stick or dowel, and a tape measure.

- Pound a stick or dowel into the ground. Using a metre ruler may make the calculations more straightforward.
- Measure the length of the stick above ground and then measure its shadow (refer to Figure 5). Record these measurements.
- Measure the shadow cast by the tree. Record this measurement.

Some suggestions on what can be used to measure the height and shadow length instead of a stick or metre ruler are: a small tree (small enough to be measured easily by the student), a pen or pencil, or a ruler. Discussion can take place about the accuracy of the shadow measurement for smaller objects, like the pen.

Figure 5. Looking at the shadow cast by the stick.

The estimated height of the tree can be calculated using knowledge of ratio and the special case of similar triangles. The ratio of the shadows should be the same as the ratio of the heights of the objects. For example, if the stick's height above ground is 100 cm (1 m) and its shadow is 40 cm, the ratio of shadow to actual height is

40 : 100 or 4 : 10

If the tree's shadow is 200 cm (2 m) long the ratio of shadow to height will be the same,

200 : height or 4 : 10

Because 4 × 50 = 200, the same factor will give the height: 10 × 50 = 500. Hence

height = 500 cm or 5 m.

Data representation

Students can use a table to record their results.

Thinking about the mathematics

What was the ratio used to determine the height of the tree? How does knowing the length of a tree's shadow help us determine its height? How does ratio help us determine the height of a tree? How do these estimates compare to the ones collected in earlier investigations? What else could we use instead of a stick to obtain a ratio between height and shadow length? Does this investigation depend on the time of day the measurements are taken? Why or why not?

This investigation uses similar triangles, as demonstrated in Figure 6, to help measure the height of the tree. As the two triangles

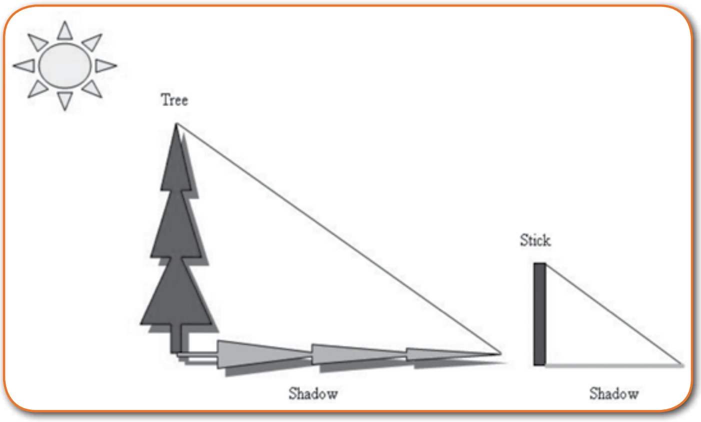

Figure 6. Using shadows to create similar triangles.

Student	Stick Shadow Length	Stick Length *(above ground)*	Ratio Length of Shadow: Actual Height *(for the Stick)*	Tree Shadow Length	Tree Height
Louise	40 cm	100 cm	40 : 100	200 cm	500 cm

created by the shadows are proportionally the same (they both have the same shape but are different sizes), the ratio found by comparing the height of the stick to its shadow length can be used to find the height of the tree compared to its corresponding shadow.

Variation in height estimates

In the *Australian Curriculum: Mathematics*, variation is introduced in Year 3 when students are encouraged to look at the variation in results of repeated trials (ACARA, 2010, p. 22). The authors contend that these investigations provide an excellent opportunity to explore the concept of variation in a measurement context. Not only can students explore the variation present within each investigation, they can also compare the degree of variation between the three methods. For example, students can be asked to determine which of the three methods had the most or the least consistency among students' estimated heights. This can be achieved through the use of technology, as discussed in the following extension activity, which allows students to gain an appreciation of variation and measurement error by enabling them to manipulate and use a larger set of estimates than just their own collection.

Extension activity using technology

TinkerPlots (Konold & Miller, 2005) is an educational software package that provides a constructivist environment for students to create graphical representations of data sets (Fitzallen, 2007). Watson and Wright (2008) illustrate the use of *TinkerPlots* in another measurement context based on students' arm spans and heights.

Using *TinkerPlots* it is possible for students to enter the estimates from each investigation, both from themselves and their classmates, into data cards. These can then be used to investigate the degree of consistency within and between each measurement technique. Figure 7 shows the possible format of the data cards. For Investigations 2 and 3, the *TinkerPlots* formula function can be used to calculate the height of the tree, as shown in Figure 8.

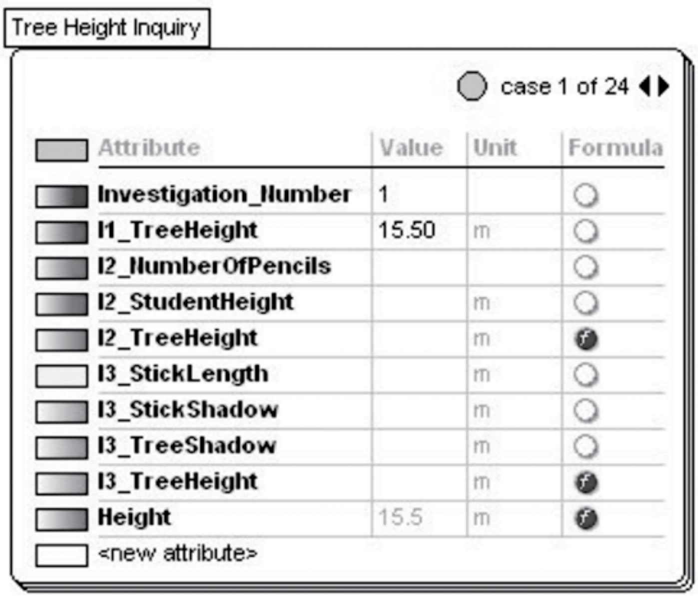

Figure 7. A TinkerPlots data card.

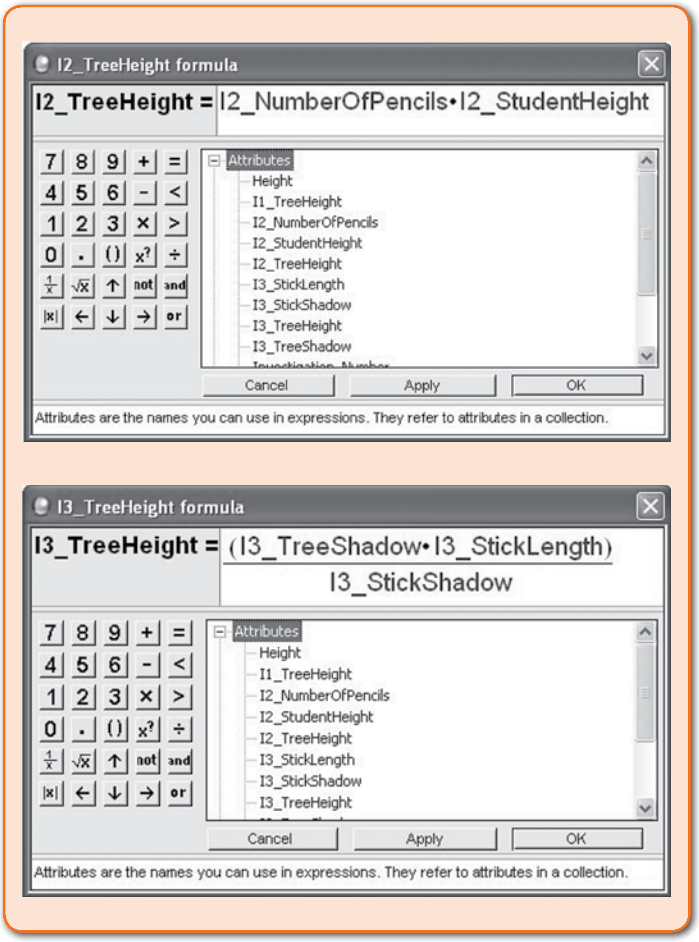

Figure 8. TinkerPlots formula boxes for calculating the estimated height of a tree.

This extension activity relies on some or all of the class estimating the height of the same tree for each of the three investigations. Once the data have been entered, it is recommended that students use the formula function and an "if statement" to create a new attribute for *Height* as shown in Figure 9. This attribute can then be plotted and students can use the mean and median tools, as well as the *Hat Plot* function, to determine the degree of variation in the estimates for each and among the three investigations (for example, refer to Figure 10).

Discussion

The purpose of this article has been to motivate teachers to present their students with meaningful investigations that lead to an appreciation and understanding of a variety of ways to estimate the height of an object that may not be able to be measured directly. The hands-on investigations allow students to make connections between different mathematical concepts, such as ratio, proportionality, and the properties of

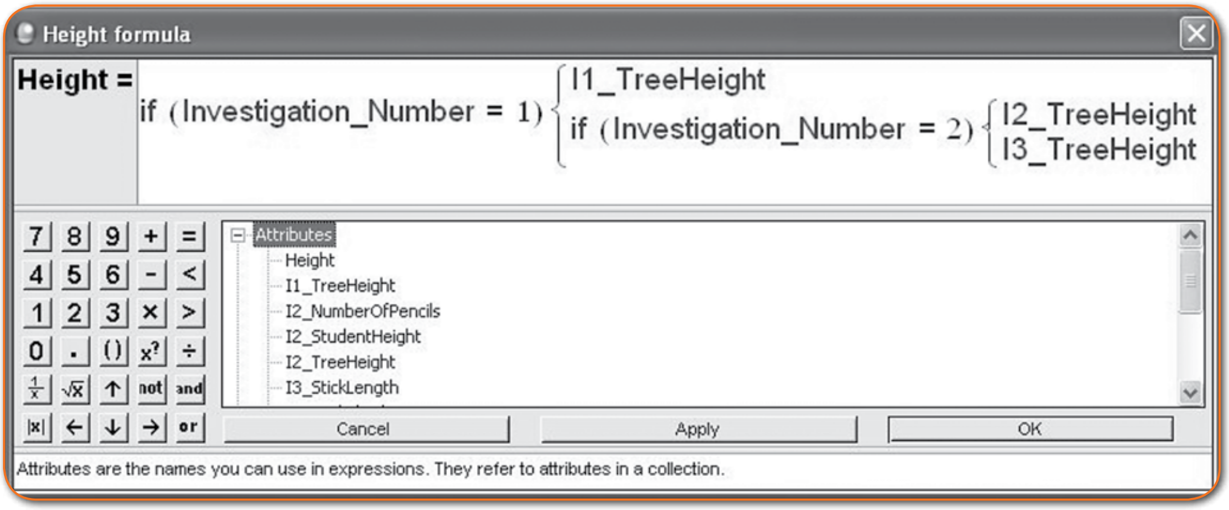

Figure 9. A TinkerPlots formula box for creating the attribute Height.

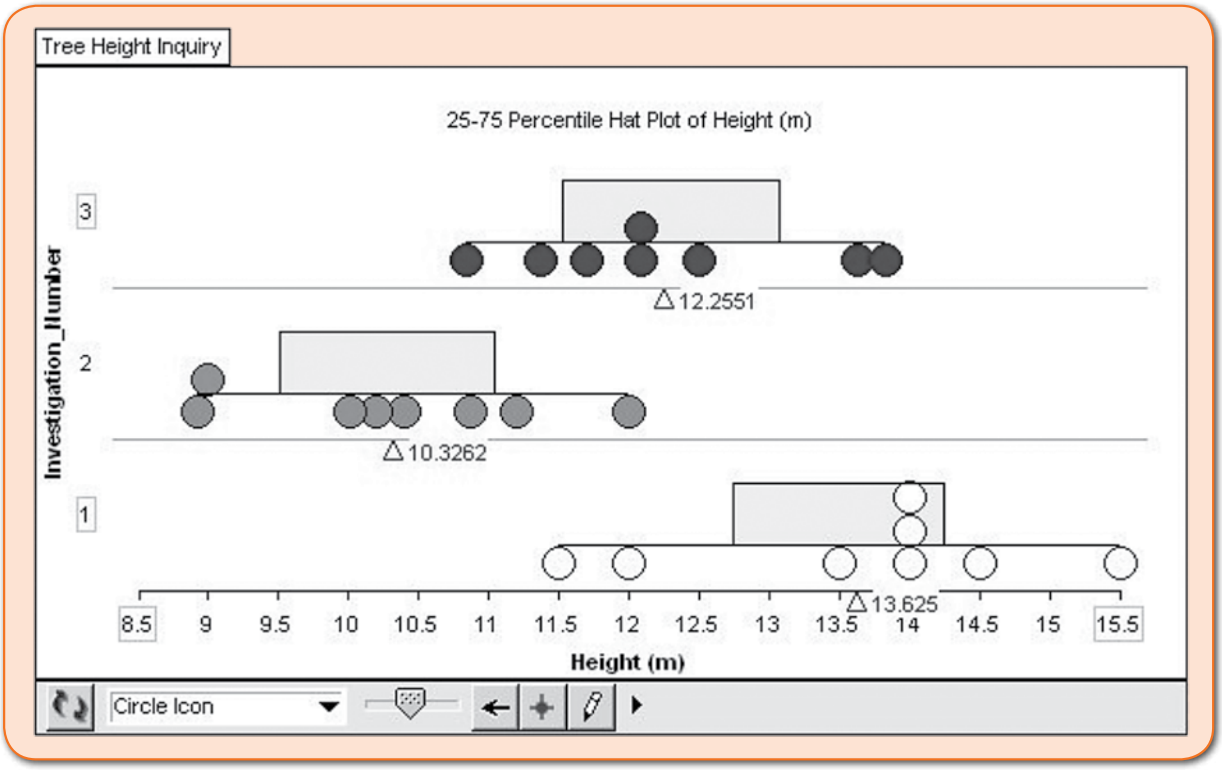

Figure 10. A TinkerPlots stacked dot plot showing the spread of values and the mean height for each investigation.

triangles, and to use their prior mathematical knowledge for an unfamiliar and "non-routine" problem—ideals endorsed by the *Australian Curriculum: Mathematics* (ACARA, 2010). The use of technology in the extension activity enables students to explore the concept of variation in measurement and build upon their understanding in this area. Extensions to this activity for middle school students using two other methods are presented in *The Australian Mathematics Teacher* (Watson, Brown, Wright, & Skalicky, in press).

Acknowledgements

The MARBLE project was supported by Australian Research Council grant number LP0560543. Key Curriculum Press provided *TinkerPlots* to the schools in the project.

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