Students,

Mathematics

& Graphics

Calculators

into the

New

Millennium

The collected papers and ideas of the
Conference held 30–31 March 2000

Students, Mathematics and Graphics Calculators into the New Millennium

Edited by Will Morony and Max Stephens.

Preface

The issue of the role and best uses of graphics calculators is one of the key challenges facing school mathematics in Australia and around the world. This volume brings together the knowledge and insights of many of this country's leading mathematics educators. It is the best advice available and will inform practitioners, researchers and other educational leaders as they seek to enhance the teaching and learning of mathematics in our schools.

The first section of the proceedings is the Conference Summary. Participants spent considerable time in small group discussions and their input on a range of key topics has been brought together in this Summary. The two major Plenaries follow. The Opening Address was given by Mr Vince Geiger, President of the AAMT — a person with great knowledge in the area. His paper helps frame the conference and set the context. Barry Kissane's clear and provocative analysis of the issues in assessment follows. Eight Master Classes were the means used at the conference to demonstrate cutting edge theory and practice and the collection of papers — along with thoughtful summaries and reflections from Discussants — is testimony to the great work that goes on daily in Australia's mathematics classrooms. The volume concludes with the collection of Shorter Presentations, all of which provide thought provoking input on particular key issues.

The Australian Association of Mathematics Teachers appreciates the support of Casio, Hewlett-Packard, Sharp and Texas Instruments in ensuring that representatives from all states and territories were able to attend the conference and make it a truly national event. Thanks are also due to the Executive and staff of the Mathematical Association of NSW who assisted the organisation in many ways. Finally, and most importantly the thanks of the Association, and, indeed, all those who care about the future of mathematics in our schools, are due to all participants for their formal and informal input to these Proceedings.

Will Morony & Max Stephens

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Conference Summary

Overview of background and process

Issues around the use of graphics calculators in mathematics learning — and the need for accompanying curriculum change — have emerged, and are being dealt with in various ways in educational jurisdictions around Australia over the last decade or so. The Australian Association of Mathematics Teachers has adopted the role of facilitating the sharing of experiences and the formulation of advice based on the collective input of those participating in the conference Students, Mathematics and Graphics Calculators. The Conference was held in Sydney on 30–31 March 2000. This Summary is the result of conferees' discussions and input. It is designed to inform the community of professional mathematics educators.

The Summary does not contain a consensus position reached by the conferees. Rather, it is nothing more or less than well informed advice to the education and wider communities on this issue. Education authorities, professional groups (including the AAMT), schools and individual teachers are invited to consider this advice in their considerations of the issues as they relate to their setting.

Clearly, personal computers can provide students with greater mathematical power and facility. This would result in some advantages if all students had access to computers with appropriate software and peripherals (rather than graphics calculators) as needed. The reality is, however, that graphics calculators are cheaper to buy, more portable and require less sophisticated management than computers. This makes them a more feasible solution in most situations.

The consideration of a fairly narrowly defined area such as graphics calculator use is necessarily done in a number of broader contexts. The sections which form the body of the Summary acknowledge particular contextual factors in dealing with areas such as mathematics curriculum, learning and so on. It is also important to acknowledge some key factors in education generally which have had impact on conferees' discussions. They are included here to indicate some of the factors which need to be thought about when considering the use of graphics calculators.

Some of these broader factors relate to mathematics in schools:

- students need to begin to develop skills and healthy attitudes to using technology for mathematics throughout their schooling, beginning in the primary years;

- emphasis on graphics calculators is one aspect of the broader directions in the use of information technologies in and for mathematics;

- there seem to be continuing public perceptions of mathematics which effectively discourage many students, and which work against many of the improvements mathematics educators see as necessary;

- there are significant concerns about the supply, recruitment and retention of well qualified teachers of mathematics;

- the capacity of some hand-held devices to perform sophisticated algebraic manipulation will profoundly challenge thinking about the nature of school mathematics and affect future developments in mathematics curriculum.

Other factors relate to school education generally:

- increasing devolution of authority to the individual school level in the public sector;

- a trend for education authorities to outsource provision of professional development and other services, a trend seen by many as being associated with a significant decrease in actual resources;

- a greater emphasis on accountability, sometimes through means which are educationally problematic;

- efforts to enhance the professionalism and status of teachers and teaching.

The Summary contains seven inter-related sections. Each contains a discussion of background, identification of key issues and highlighting directions that may need to be considered. The contents of each section were provided by the written records of all the small group discussions at the Conference. Conferees have had the opportunity to comment on a draft before its finalisation.

1. Curriculum

Discussion

Use of graphics calculators has an impact on teaching, content (including sequencing of content), learning and assessment. Graphics calculators should not drive the curriculum but changes will occur as the result of research into how students learn, and as relevant subject matter becomes more accessible. Most content areas will remain broadly the same, but tasks within a given area will reflect the availability of graphics calculators. It becomes more feasible for students to move from the 'particular' to the 'general'. If graphics calculators are viewed as tools for learning, teaching methodologies, too, will change.

Good curriculum design — at the system and school/individual teacher levels — will take advantage of graphics calculators in relation to all of these, while setting high expectations for students' learning. It supports teachers in exercising flexibility of delivery appropriate to their students.

Systemic advice about graphics calculator use in teaching and learning, and decisions relating to assessment have to be made in the interests of the majority of schools, teachers and students. Successful planning for any changes will involve the relevant educational authorities (curriculum councils, boards, etc.) being clear about and remaining faithful to principles and workable timelines for consultation and any eventual implementation.

It is an agreed principle that students must continue to focus on important areas of mathematics. For example, the shape of graphs of functions, including key features such as axis intercepts, location of critical points, and so on remain fundamentally important. Many would argue that the effective use of graphics calculator technology both depends on and should also extend students' appreciation of these fundamental features of functions and

their graphs. But whereas these aspects in the past needed to be approached through careful graphing of points combined with algebraic techniques (some of which are difficult), these key features can be identified more readily and clearly, and explored first. Features of families of curves can be identified and drawn together. Analytical exploration of these features remains important but no longer needs to be done as the starting point. The same applies to introducing more complex functions to illustrate growth and decay, or effects of damping, and to other areas of mathematics.

The above discussion illustrates an aspect of mathematics content that remains important and essentially unchanged. Other areas such as treatment of domain and range of standard functions and modifications of these functions actually become more important. On the other hand, topics and techniques which in the past were the only methods available to students may require less emphasis or even become optional. For instance, the technique of manually completing the square to determine the turning point of a quadratic is now only one of several techniques that pre-calculus students should be exposed to.

Issues

A key factor in successful implementations of graphics calculators has been the development of strategies which foster leadership at the school level, help principals and administrators to make well informed decisions for schools, parents and students, and above all, to engender confidence in classroom teachers. Identifying, evaluating and implementing these kinds of strategies remains an ongoing challenge.

A direct result of the fact that technology — including graphics calculators — continues to evolve is that it is not sensible to specify the actual types and uses of technology in narrow ways through curriculum guidance and syllabus documents.

Teachers need to emphasise the distinction between calculator results and sound mathematical reasoning and generalisations. Wherever students use graphics calculators, their work needs to exhibit sound mathematical expression, argument and even proof. There may also be changes in the requirements for showing work and setting out mathematical argument.

Directions

- Effective implementation may include a staged introduction for different courses, subjects or year levels in order to diminish issues of resource provision and to promote effective in-service support for teachers.

- Implementation of new curricula involving graphics calculator use will rely on schools and teachers having access to sample courses/programs/syllabuses incorporating use of graphics calculators, including sample assessment tasks appropriate for different year levels and for different courses.

- Those responsible for developing curricula and syllabuses have a particular responsibility to develop materials and processes which give clear guidance about the spirit and intention of new syllabuses rather than merely leave interpretation to text book writers. There needs to be clarity about syllabus writers' intentions for the kinds of things they expect students and teachers to be doing, and text book writers need to consider and act on this information.

- Resources and professional development programs must enable teachers to assist their students to distinguish between appropriate/ inappropriate and effective/ ineffective uses of technology in order to emphasise this intention of the curriculum.

- Materials and support should demonstrate the interplay between use of graphics calculators, sound conceptual development and analytical approaches. Intelligent use of any mathematical technology can only be underpinned by sound mathematical knowledge.

- Access to graphics calculators allows existing content to be freer from limitations (non-integer coefficients for quadratic and other functions are easier to work with), current topics to be extended (for example, combinations of functions), new topics and techniques to be introduced, changed emphases in some content areas, and a decreased emphasis in other areas (one such area might be logarithmic solutions to exponential equations). Other areas where change is likely to occur are numerical considerations of rates and solutions of differential and other equations, iterative procedures, fitting curves to real data (to an appropriate degree of accuracy), and tvm (time/value/money) financial problems. The use of associated peripheral technology allows students to gather and work with real data. Given that students can work with data and a wider range of functions, there is likely to be a greater emphasis on application questions which involve more complex modelling situations. As applications and modelling tasks are introduced, there is clear need to consider appropriate degrees of accuracy and the purposes of creating and using mathematical models.

- Access to graphics calculators allows new possibilities for sequencing of, and within, topics, e.g. graphical approaches can be utilized more effectively to precede analytical work, and can be used to provide a stronger foundation for mathematical understanding. As a consequence, there may well be some refinement of content in various topics, such as numerical solutions of polynomial and differential equations without readily accessible analytic solutions. In other areas, the order of treatment of a topic may change; for example, maxima and minima questions and optimization problems can be introduced before calculus is developed.

- Access to graphics calculators allows alternative approaches to introduction and development of key ideas, for example, through numerical, graphical and symbolic approaches; rich opportunities for problems to be considered in an interactive way, using, for example, a mixture of analytical and numerical/graphical techniques to form and test conjectures; heightened awareness of 'exact' and 'approximate' solutions; increased student independence in accessing correct results to work with and forming and testing conjectures. Teachers must employ specific strategies that enable students to learn when a task can be equally well addressed by a graphics calculator or by an analytical approach, and when an analytical approach must be used to establish a particular or general result or product, rather than using an empirical approach alone. Inappropriate use of technology is likely to lead to bad mathematics.

- Access to graphics calculators allows greater independence for learners, increased opportunities for mathematical investigations and modelling problems, thereby providing contexts in which students can develop skills in mathematics writing and

expression. The process of posing, evaluating and refining conjectures requires students to draw on existing mathematical knowledge, to recognize that conjectures need to be confirmed or rejected using mathematical argument based on further testing and analysis, and that confirmation or rejection can be achieved in a range of ways. Graphics calculators can add significantly to students' options and capacity to engage in these activities.

- If calculators are used in teaching and learning they should not be excluded from assessment. There needs to be an alignment of student use of graphics calculators in activities related to learning mathematics with corresponding use of graphics calculators in the assessment of this learning. It is essential to devise assessment tasks requiring judicious use of graphics calculators, showing a balance across types of tasks, including both school-based assessments and more formal (systemic) examinations, where these are employed. Such tasks might include calculator-free tasks, calculator-active tasks, tasks which assess mathematical understanding, and tasks which call for mathematical analysis and give appropriate importance to mathematical explanation and justification. There is also a need for items which expect students to choose when, and when not, to use their calculator.

2. Learning

Discussion

Curriculum and teaching guidance around the states and territories identifies new and emerging emphases in the learning of mathematics which are broadly similar. Among these:

- applications and modelling are recommended to enhance student engagement and demonstrate the essence and usefulness of mathematics;

- strategies which enable explicit attention to outcomes relating to thinking and doing mathematics are important;

- individual and collaborative working modes, which mirror those in work and life generally, are encouraged.

What needs to be identified and understood is the extent to which students' use of graphics calculators — as personal, hand-held technology for mathematics that is capable of symbolic, graphic and numeric representations — enables these and similar goals to be realised, and the ways in which this can be done.

Issues

The potential of any approach — in this case the use of graphics calculators — is only able to be achieved in sustained and coherent implementations. Even at the earliest stages of use in the classroom, students' learning can be enhanced (for example in terms of families of functions which graphics calculators display well). However, examples of their use in ways which promote the kinds of directions outlined above can only really come from settings in which the appropriate use of graphics calculators is routine. It is in these settings that teachers are focussed on working in ways which promote learning by taking

these directions in their work with students.

Directions

- Exploit the multi-representational interface and the dynamic capabilities of graphics calculators to help students become comfortable with different forms of the same mathematical idea, and the connections between these representations and between mathematical ideas.

- Work with 'real world' applications, in which the data may be messy but now feasible for students to use, and using these contexts not only to illustrate mathematical concepts, but also to develop these.

- Promote explicit attention to the learning of mathematical processes in addition to learning mathematical content by asking students to explain and justify, and similar questions. Students need to develop the ability to represent learning based on their use of graphics calculators to answer these sorts of questions and to communicate mathematics generally.

- Understand and promote graphics calculators as tools for communicating mathematics, communicating about mathematics and in interactions in mathematical settings generally. Specific attention needs to be given to their use in and impact on learning in individual and collaborative settings.

- Teachers have access to better understanding and diagnosis of students' conceptual understandings. These conceptual understandings, rather than the mechanics of obtaining a solution, can provide more of the focus of the teaching and learning processes when graphics calculators are used.

- Exploit the graphics calculator as 'personal assistant' in ways that encourage student curiosity and enable students to explore mathematical ideas and applications.

- Encourage and enable mathematics teachers to collaborate with colleagues in other discipline areas that use graphics calculator technology to create consistency of approach and expectations, and to share tasks.

- Establish the ways in which the use of graphics calculators can promote and enhance human-centred views of and approaches to the use of technology more generally.

- Scaffold students' progress in learning by providing different learning pathways, both in terms of different learning styles and responding to gaps in students' prior knowledge.

3. Assessment

Discussion

There is some argument that separation of 'assessment' from 'learning' is artificial and sends inappropriate messages. Certainly the principle of alignment and consistency between teaching and learning on the one hand, and assessment on the other, underpinned discussion of assessment at the Conference. There are, however, unique and important issues around graphics calculators and assessment. These emerge in the classroom where teachers largely control the assessment processes. They are especially evident in high stakes external assessment. Hence a section of this Summary is dedicated to assessment. Also, conferees took the view that assessment needs to be dealt with positively and constructively in the change processes associated with the implementation of graphics calculators. Assessment needs to take advantage of graphics calculator use, rather than merely accommodating it.

The relevant authorities in Victoria and Western Australia have adopted a policy of assuming access to graphics calculators in courses with high stakes assessment, including externally set examinations. This has led to consistent, high usage patterns by students in these states. In this context of universal use, teacher competence and confidence in using graphics calculator technology has emerged as a major equity issue for students.

Most of the matters canvassed at the Conference and outlined below related to assessment generally, and not specifically to high stakes externally set examinations — this context does serve to bring the issues and challenges into sharp focus, however.

Issues

There is, at present, no single pathway for graphics calculator use in formal assessment. Approaches including calculator free sections, calculator neutral questions, items in which calculator use is expected are being tried as people try to match curriculum purposes with the ways students are assessed.

It is acknowledged that the advent of Computer Algebra Systems (CAS) on hand-held devices will continue to challenge notions of what mathematical knowledge is valued and how students' skills and understanding can be assessed, given the power that is, potentially, in students' hands. This is particularly an issue for assessment regimes associated with centralised curricula.

At the practical level there is the issue of controlling calculator capabilities. Capabilities provided through the use of 'flash memory' can only be determined by actively using the calculator; using small programs can make less powerful calculators equivalent in capability to others in which the function is integral, but will be lost if the memory is cleared for some assessment purpose.

Directions

- Find and explore ways to broaden the scope and complexity of possible tasks, including tasks which provide a variety of pathways to appropriate solution(s).

- Ensure that assessment practices involving graphics calculators provide tasks across a wide range of difficulty levels.

- The practice of allowing students to take several prepared pages of their own notes (e.g. two A4 sheets) into examinations can defuse problems which authorities have with allowing calculators with memo/notes facility.

- Establish norms for students to provide an adequate working trail that enables tracking of their work and allocation of part credit.

- Develop means for assessment which enable evaluation of the student's critical and informed use of their graphics calculator, and their ability to choose (or not) to use their calculator.

- Establish and maintain processes which enable teachers to share assessment materials (examples, student work samples etc.) as a high priority to avoid 'reinventing the wheel'.

- Encourage and enable teachers and others to continually explore new territory in assessment by establishing a culture of valuing and publicising this work.

4. Teacher support — teaching and learning resources, training, professional development

Discussion

The need to provide effective support for teachers was consistently identified as a major — perhaps the key — ingredient in exploiting graphics calculators to enhance school mathematics. It was a theme which recurred in many ways.

The implementation of graphics calculators for school mathematics learning requires identification of stages in teachers' knowledge, understanding and skills, and needs to assume that teachers have sound knowledge of the mathematics they teach in order to become intelligent and powerful users of graphics calculators for teaching and learning. Strategies need to be developed for the provision of training and professional development, and for making available teaching and learning resources which are tailored to these needs. The stages might be characterised as:

- novice, or beginner, in which the needs are functional in terms of personal, technical skill with the graphics calculators, with some classroom activity when the teacher feels comfortable to do so;

- practitioner, in which teachers' fluency with graphics calculators enables them to draw its use into a range of teaching and learning settings. Appropriate teaching resources and professional development to particular curriculum areas are important in this widespread classroom implementation;

- creator, in which teachers and students explore new teaching and learning territory with graphics calculators as a natural tool and vehicle for their work. It is these settings in which innovative best practice develops and can be observed.

Conferees acknowledge that a great deal is known, in general, about teacher professional development. Good practice in professional development is expected in programs developed and implemented. The comments and suggestions below consider and apply this

general knowledge to the use of graphics calculators in particular.

Issues

A major consideration is that of funding provision and the extent to which teachers actually have the opportunity to participate in professional development and training. The extent of funding from employing and accrediting authorities, and particularly the ways it is provided have changed. It is a context in which strong reasoned argument for funding and support is needed at the national, state/territory and local levels.

Working 'smarter' to provide teacher support is essential. This includes effective collaboration between systems, schools, teachers and their organisations and corporate partners to build on valued existing support as alternatives to the more traditional funding sources. It is important that programs sponsored by graphics calculator companies continue to go beyond dealing with the 'novice' stage. Coordination of these and other efforts is another important element. Local, national and international teacher networks are very effective means for sharing resources and information. This is important at the practical level for 'working smarter'. It is also an indicator of a mature profession taking charge of its own directions.

Coordination of effort is made possible if any moves to the required or assumed use of graphics calculators provide an adhered-to timeline that allows enough time for teachers to acquire necessary knowledge and skills. The amount of time that relevant authorities need to plan for and provide for teachers should not be underestimated.

Directions

- Careful attention to the design of starter materials for teachers and students that meet initial needs but encourage further growth.

- Exploit the use of the Internet, in addition to more traditional means, for materials and PD delivery, and for networking and sharing of resources by teachers. Initiation and coordination of this was signalled as, potentially, an AAMT responsibility. Sharing experiences enriches those on their way as well as providing evidence of what is possible for the less experienced.

- Particular effort is needed to ensure effective in-school leadership (head of department and/or one or two enthusiasts) and work to a local plan that is self-challenging and self-sustaining.

- Develop new genres of teaching and learning materials that go beyond 'worksheets/activities/blackline masters' to those which provide resources that integrate calculator use. In particular the new style resources and ways of doing things need to focus on the mathematics and learning independent of the particular graphics calculator being used.

- Further employ good practice in professional development embodied in such things as effective mentoring programs, the Maths in Schools initiative — this involved schools (in partnerships with the Mathematical Association of Victoria and University staff) driving professional development programs that met school identified needs — as well as programs which integrate materials and professional development.

5. Equity — provision of graphics calculators for students to use

Discussion

Ensuring that students have access to graphics calculators is a concern in those jurisdictions which currently do not expect their use. However, in Western Australia and Victoria it is a much smaller concern. In practice, all students do have access for learning and assessment since this is a requirement of their courses.

It is a commonly held view that the most effective approach is for students to have open and continuous access to graphics calculators for learning inside and outside formal mathematics time. This implies that they have their 'own' calculator, rather than the use of 'class sets' returned at the end of the lesson — although this may be appropriate in the early secondary years when students are being introduced to the use of graphics calculators. Many teachers also report that it seems more efficient if all students in the class use the same model of calculator.

While not necessarily easy, the matter of provision of graphics calculators for students seems able to be resolved. It has been noted that sufficient calculators for a class can be purchased for little more than the cost of a single personal computer. Once all students have appropriate access to graphics calculators, the focus needs to be on addressing the differential skills of teachers with the technology. Students in classes with teachers who are not well-versed are disadvantaged in comparison with others who are taught by teachers who are. Ensuring that all students have access to graphics calculators is necessary, but it is the advantage that is taken of it in the learning process that makes a difference for students.

Issues

Requiring students to have access to graphics calculators (effectively mandating their use) has been resisted on the grounds of equity — less well-off schools and students are seen to be unable to afford to get them. Leaving their use as optional, however, means that the least well resourced people are least likely to have access to them — there are other more demanding calls on scarce resources. Hence, effectively mandating calculator use can enhance equity.

A major issue is to ensure that graphics calculator use for mathematics is seen by education systems and schools as part of general efforts to implement Information and Communications Technologies (ICTs) in school education. Increasing use of ICTs is a priority in education around the country, but is often seen as exclusively meaning use of computer technologies. Mathematics educators need to mount effective arguments to systems and school councils to gain this recognition for graphics calculator technology. Such recognition will be made tangible through the funding necessary in relation to both the provision of the technology (if that is an issue) and ensuring that all teachers can and do undertake the specific professional development they need to be effective in their classroom use of graphics calculators.

A practical issue arises in settings with senior (Year 11–12) colleges and multiple feeder schools (Tasmania and Australian Capital Territory in particular). Students entering the senior college may have different calculators, depending on the preference of their high school. Classes in which students use a number of types of calculator can create extra diffi-

culties for teachers, and may serve to discourage those new to using them in their teaching. Enforcing a particular calculator choice on feeder schools is probably not feasible, however. Equally, it may not be educationally appropriate to insist that students do not use graphics calculators until senior mathematics. This is also an issue when students transfer between schools.

Directions

- Education authorities and schools need to implement strategies which enable students to have access to the technologies appropriate for their learning of mathematics in ways that maximise the benefits to their learning.

- Similarly, there needs to be support in the form of teaching and learning resources and provision of teacher professional development to ensure teachers have the skills and knowledge to make effective use of graphics calculators to enhance student learning.

- From a base of being 'at home' in their use of technology generally, teachers and lecturers need to work to become sufficiently proficient to be able to assist students using any of the major brands.

- Some of the models of provision that have been used to provide physical access include:

  — schools purchase class sets with added incentives from companies;

  — schools buy in bulk and on-sell to students;

  — arrange for reselling of used calculators (whether formerly owned by the school or other students) to younger students;

  — use of leasing plans.

6. Post secondary (including teacher pre-service)

Discussion

Although not a key focus of the Conference, issues around students' post-secondary experiences were raised. These related to undergraduate mathematics and other courses as well as to direct mathematics teacher preparation. It is accepted that secondary and tertiary education are different contexts and the students have different needs.

Any discussion of the tertiary sector needs also to acknowledge the context that universities can be seen to be in a state of flux with issues of changing clientele, changing expectations and uncertainty about structures and staff continuity.

Issues

It is also acknowledged that students make a great deal of positive and effective use of technology in undergraduate mathematics courses. There appears, however, to be little familiarity with the potential for graphics calculator use among tertiary mathematics staff.

In relation to graphics calculators in particular, it is clear that more students (particularly in Victoria and Western Australia) are entering university as very competent users of graphics calculators for doing and learning mathematics. Continuity and capitalising on what these students can do is a worthy goal, but one which cannot be driven entirely from the secondary schooling end.

A practical issue for teacher educators is that of ensuring that graduating teachers have considered the range of issues associated with graphics calculators. They should have sufficient familiarity with the use of graphics calculators and have appropriate skills to enable them to enter a teaching workforce in which such skills are assumed.

Directions

- Find ways of engaging tertiary 'mathematics oriented' staff and building awareness of graphics calculators and their potential for mathematics learning.

- Teachers and the school mathematics community can engage with tertiary colleagues most effectively by working in partnership with professional societies of colleagues in the tertiary sector.

- Explore ways of creating greater coupling of practice and understanding between undergraduate mathematics teaching, learning and outcomes and mathematics teacher education.

- Encourage and facilitate networking of those working in mathematics teacher preparation in relation, particularly, to the use of graphics calculators in the secondary classroom.

7. Research

Discussion

Continuing research effort is essential to inform the actions of mathematics educators and others (administrators; curriculum and assessment authorities) in relation to graphics calculators.

Action research and 'best' practice were identified by participants as the preferred models for research and development in this context. Both these emphasise a 'teacher as researcher' orientation. In the context of graphics calculators, a great deal of the leading edge work to date has been done by teachers in classrooms. In many respects the practice has got ahead of the research. While this is a strength in terms of the practicality of findings, it can result in an emphasis on anecdotal evidence and assumptions. Hence, appropriate incorporation of 'balancing' formal research methods is important. The role of this formal research can be seen in part as identification, distillation, analysis and synthesis of the practice in order to feed this back to the practitioners.

Issues

Although it is acknowledged as important that educational change is guided by research, it is not feasible, to expect to develop a complete and coherent research base before taking any action. What is possible, and what should be the focus, is a continuing program to

research and identify the ways in which graphics calculators can enhance mathematics learning for the citizens of the future and enthuse students in relation to further learning of mathematics.

An ongoing issue in studies which try to identify 'effectiveness' of a particular teaching approach (such as graphics calculator use for mathematics) is that the teaching occurs in a particular context. Hence it is a question of whether, and how, to distinguish the effects of the general context from the actions specifically in relation to graphics calculators, and, potentially, to do this for different groups of students.

A general issue for research in education is to develop and implement strategies for the sharing of the research findings across systems and states/territories.

Directions

In order to build on the strengths of the current situation, a genuinely collaborative effort between school practitioners/researchers and tertiary researchers is needed. The contributions of each partner need to be equally respected and considered.

Following is a listing of potentially rich research questions/issues.

- In what ways does graphics calculator use improve conceptual understanding and, if so, does this result in improved retention and transfer of mathematical ideas and processes?

- What are the effects on students' problem solving skills? What can these effects be?

- How does graphics calculator use affect the strategies students use?

- How does graphics calculator use affect the difficulties and misconceptions that emerge in students' learning?

- What factors affect students' capacity to choose when, and when not, to use a calculator?

- To what extent does the use of graphics calculators produce differential effects for different users ('ability', gender, socio-economic status, preferred learning style)?

- Studying and understanding the processes of change and implementation in relation to graphics calculators.

- Issues in the curriculum — construction, staging of content, development, implementation, evaluation.

- Integrating and highlighting research that is classroom based or based on professional development.

- Interrelationships between graphics calculators, curriculum/assessment, pedagogy, mathematics and students.

- What are the different ways in which students with different learning styles use graphics calculators?

- The effects of graphics calculator use on classroom interactions and dynamics (student-student, teacher-student).

- Strategies for teaching and learning when students use a variety of calculator types in the one class.

- How can there be coherence and consistency in graphics calculators use across the curriculum?

- How classroom use of graphics calculators changes what teachers do and how they do it?

Plenary Addresses

Background and Issues for the Implementation of Widespread Access to Graphics Calculator Technology

Vince Geiger

President of AAMT
Hillbrook Anglican School, Queensland

Background

The call for the inclusion of technologies such as computers and graphics calculators as a necessary adjunct to the teaching and learning of mathematics has been echoed in the literature and curriculum documents for at least the last decade (e.g. Barrett & Goebel, 1990; Demana and Waits, 1990). The potential for technology to enrich mathematics curricula, teaching, and learning has been noted in documents such as the NCTM Standards (National Council of Teachers of Mathematics, 1989, 1991), which explicitly advocate the use of technology in secondary mathematics classrooms. In Australia it is similarly recommended that all students have ready access to appropriate technology as a means to support and extend their mathematics learning experiences (Australian Association of Mathematics Teachers, 1996).

Despite long term attention, the use of computers and other technologies in school mathematics classrooms has been restricted, until recently, by economic, social and practical constraints (Kissane, Kemp and Bradley, 1995; Kaput 1992). At the same time other commentators have insisted that 'to expect that schools and teachers can continue to exist apart from serious technological support is hopelessly myopic' (Kaput & Thompson, 1994, p. 682).

The circuit breaker to the tension between the need for the students and teachers to have free access to technologies, and what schools are able to provide, appears to be the development of affordable hand held devices which have become known, variously, as graphing, graphical and graphics calculators. A number of reasons have been put forward for the now favourable disposition of a growing number of Australian school systems toward the use of these devices. However, their increasing acceptance appears to be strongly related to the very good value for money they offer in relation to computing power verses cost when compared to a computer equipped with appropriate mathematics specific applications. Graphics calculators represent a one stop shop with respect to the type of facilities presently required by a typical secondary mathematics student at a price point where a class set can be purchased for around the price of a single computer.

With increasingly widespread access to graphics calculators we are at a point in this evolution where it is important to map the landscape and to identify issues for curriculum,

teaching/learning practice and assessment. It is a time to share developments and ongoing considerations at the state and territory level with a view to sharing a vision for the future.

Australian teachers have a growing reputation, internationally, for being at the forefront of creating new knowledge in this area. This, in turn, informs creative and innovative best practice. The reputation of Australian teachers is based on the quality of their thinking and willingness to 'break the mould' when they act to implement change in their classrooms. Teachers have been working with graphics calculator technology well before it was sanctioned by their various state systems for use in high stakes examinations. It is the work at this practical level that has provided the support needed by future orientated policy makers to bring about system wide reform. The introduction of graphics calculator technology is a significant example of how teachers are acting as researchers in their own classrooms, not only to understand accompanying change, but also to inform their role as leaders in, and managers of, innovation.

The Australian and the broader context

The growing acceptance, in Australia, of the need to provide students with access to mathematically enabled technologies, at all times, is evidenced by changes in policy with respect to assessment practices in each of the states and territories. Students in Victoria (1997) and Western Australian (1998) now have assumed access to graphics calculators in the Victorian Certificate of Education (VCE) and Tertiary Entry Examinations (TEE) respectively. South Australia has also declared its hand by notifying schools, this year, that there will be assumed access to technology in Senior Secondary Assessment Board of South Australia (SSABSA) exams from 2002. Queensland has always had a policy of non-mandatory unrestricted access. Other states and territories are still considering their position.

There is no clear direction in this area internationally, but there is a growing interest and acknowledgment of the potential of technology to enhance and extend what students can know and do in mathematics. In the UK, graphics calculators have been permitted in A level examinations since 1994 although papers are split into non-calculator and calculator permitted parts. Graphics calculators have also been allowed in Advanced Placement Calculus examinations in the USA since 1993.

In France any technologies below a certain physical size are permitted. This means that graphics calculators, even those that incorporate Computer Algebra Systems (CAS) are allowed in the final high school exam, the Baccalaureat. Portugal recently made graphics calculators compulsory.

In Austria, the ministry of education equipped all schools with *Derive* in 1992, and now have the largest number of TI-92s in use, relative to population. In the vocational high schools, around 50% of the students use the TI-92.

The situation in Germany is very much like ours in Australian where each of their 16 states has its own policies. In some states use of graphics calculators that incorporate CAS is mandatory, while in others, such as Bavaria, general high schools still ban even programmable calculators.

The use of graphics calculators in schools in the Asia-Pacific region has been limited to

date, although there is developing interest, to varying degrees, in Hong Kong, Japan, Singapore, South Korea and Thailand. At this stage only Singapore has declared an intention to permit the use of graphics calculators in their Further Mathematics 'A' level examination starting from 2002. Further Mathematics is designed to meet the needs of students who are tertiary bound and take this subject as preparation for courses such as engineering.

While these changes may represent first steps toward a change in attitude in relation to the use of technology in the teaching and learning of mathematics, they must also be viewed with some caution. It should be noted that most of the more successful Asia-Pacific nations in the recent Third International Mathematics and Science Study (TIMSS) do not provide their students with access to technology during high stakes assessment. This should not invoke the simplistic response that they are doing better because they delay their students' access to mathematically enabled technologies as we have no data on how these students would have performed if their systems had permitted access. It must, however, challenge any assumptions that access to technology will automatically result in improved performance or that widespread access and use alone will make us world leaders in mathematics teaching and learning. This is an issue that must be closely monitored.

Present and emerging technologies

There has been a rapid development in the number of features offered by graphing calculators and their computational power. Ten years ago at the AAMT biennial conference in Hobart, Barry Kissane (1990), in one of the major presentations of the conference declared:

> *Graphing calculators are a relatively new invention, having been in the market place for about five years. This provides a substantially changed way of doing many mathematical things and may well change the way students do mathematics.*

He was referring to devices, available at that time, such as the Casio FX-7200G and the Sharp EL-9000. Rumours were also rife that Texas Instruments was about to release a graphics calculator that was aimed specifically at the educational market. These early graphing calculators had the capacity to plot elementary functions, to solve equations numerically, operate with matrices and perform uni-variate and multi-variate statistical analyses on data sets of a reasonable size. It might also be noted that Hewlett-Packard also had available the HP-28S, which included the above mentioned features and an early CAS, although this calculator was not developed to meet the needs of the school education market.

Contrast these capacities with those of the array of devices now available. Top of the range calculators include such features as: CAS facilities (TI-92, TI-89, Casio Algebra FX 2.0, HP 49G), dynamic geometry (TI-92), hard-wired tutorial systems (Casio Algebra FX 2.0) and 3D and differential equation plots (TI-92 plus, HP49G). You can now expect a mid-range calculator to include financial and inferential statistics modules, as well as the capacity to manipulate complex numbers. Within this range, however, you might also acquire such features as a dynamic graphing module and a system of equations solver (Casio CFX-

9850GB), or a function based slide show and touch screen (Sharp EL 9600).

HP-38G also introduced the idea of making extended use of small, concept-specific programs, known as Applets, available through their website and downloadable into the calculator. This concept of the calculator as a 'shell' technology appears to be promising but is only likely to reach its potential if and when 'flash' memory becomes the industry standard. Flash memory not only allows the calculator to accommodate larger and more complex programs but also makes available the potential to upgrade the operating system of the calculator itself. This sort of upgrading capacity will obviate the need to buy a new calculator when new features become available as a user will be able to download the new feature from another source and 'soft-wire' it in.

Entry level calculators have all of the features their predecessors of ten years before, plus some additional facilities and enhancements but now for a cost in the vicinity of only $50.

Nearly all graphics calculators have the facility to operate with peripheral devices. Thus a calculator's functionality can be extended, via data logging devices, to include the measurement of distance/time data, force, heart rate, oxygen concentration and many other physical phenomena. The potential to provide students with the opportunity to measure, record and analyse this sort of data has attracted the interests of those who teach subjects other than mathematics, particularly in the physical, biological and social sciences.

Graphics calculators can typically be interfaced with personal computers to exchange programs created on either technology or to download or upload programs or data to or from an appropriate website. The capacity to interface with computers also allows the user to print out a calculator display or to take screen shots that can be placed in a document or report.

Future developments are a matter of rumour but predictable innovations include an enhanced capacity to upload and download data directly to and from the internet and the facility to share what is on an individual graphics calculator in a public fashion in group settings. Both of these features would be consistent with what appears to be an emerging trend towards the development and enhancement of the teaching/learning potential of these devices rather than merely extending their labour saving or number crunching capacities through a broader range of facilities. This is a trend that must be encouraged and supported by educators and their involvement must be seen as crucial if they are to have a role in shaping the future of this technology.

Teaching/learning

While many of the predictions of the impact of learning technologies have focused on enhanced student learning outcomes such as concept development (e.g. Vonder Embse, 1992; Jones and Lipson, 1993), and varied approaches to instruction (e.g. Dance, Nelson, Jeffers and Reinthaler, 1992), others have suggested that the most significant changes will be related to the ways students and teachers will interact in mathematics classrooms (Burrill 1992; Geiger and Goos, 1996; Galbraith, Goos, Renshaw and Geiger, 1999). Overall findings, however, concerning the 'value adding' to students' learning through the use of technology have been somewhat inconclusive (Kuchler, 1998; Lesmeister, 1996; Maldonado, 1998; Penglase & Arnold, 1996), although many studies have reported that the

use of technology has a positive effect on students' understanding of function and graphing concepts, spatial visualisation skills, and connections between mathematical ideas (Portafoglio, 1998; Weber, 1998). Further, Kutzler (1999) has argued that mathematically enabled technologies have the potential to scaffold student learning in such a way that gaps in prior learning can be managed so that they do not interfere with the acquisition of new mathematical ideas and concepts.

Penglase and Arnold (1996), in a critical review of recent research in the use of graphics calculators for learning mathematics in secondary classrooms, found that students' attitudes towards the study of mathematics generally became more favourable as a result of regular use of graphics calculators. Further, they noted that a belief among teachers in the helpfulness of graphics calculators in learning mathematics was also evident in a range of studies. The only reservation being expressed related to a fear of 'de-skilling' or loss of competency in doing certain forms of mathematics without the support of a graphics calculator.

It seems that less is known about how the availability of technology, and especially graphics calculators, has affected teaching approaches, as many studies fail to distinguish the role of the technology from that of the instructional process (Penglase & Arnold, 1996). There is a growing body of literature, however, that suggests there is great potential to change the way students and teachers interact in mathematics classrooms (Burrill, 1992; Geiger and Goos, 1996; Galbraith, Goos, Renshaw and Geiger, 1999). Some studies have found that the introduction of technology can support changes in classroom dynamics leading to less teacher centred and more investigative environments (e.g. Simonsen & Dick, 1997). It should be cautioned, however, that such changes are most likely to transpire in the classrooms of teachers who are confident users of technology and who have a predisposition towards the negotiation of shared leadership roles within their classrooms (McDougall, 1997; Simmt, 1997; Tharp, Fitzsimmons & Ayers, 1997).

Integrating technology into the teaching, learning and assessment of mathematics places significant demands on teachers to develop not only technical competence, but also what we might term meta-technological awareness (Mingus & Grassl, 1997). In other words, becoming proficient with technology requires more than facility with keystrokes and menus. In addition, teachers need to be able to recognise where and how technology can serve as a catalyst for students' learning, just as students need to learn how to choose which type of technology is appropriate for different purposes.

Although claims about the advantages to learning and collaboration are similar, significant differences exist between computers and calculators as tools for learning apart from the obvious inequality in computational and processing power. Kissane (1995) argues that accessibility and location provide significant advantages for graphing calculators over micro computers as the relatively low cost of these devices and their portability provide students with the opportunity to own and attain intellectual intimacy with these devices and thus the mathematics they learn with them. Further, because these devices are programmable is possible to customise and therefore personalise the calculator, further enhancing a feeling of ownership of the device and the mathematics it is used to explore.

Curriculum content and design

What is already under consideration in states, where access to graphing calculators is permitted, is what set of mathematical ideas and concepts constitute the 'new basics'. With the power and features now available through graphics calculator technology, the mathematical content of school syllabuses and study specifications is in urgent need of review. What we face, however, is a review that is broader in scope than perhaps any we have previously attempted.

Schools and systems have always attempted to make judgements about the relevance of what they teach in the context of the needs of society and the opportunities on offer from available technologies. Logarithms, for example, were once taught as a labour saving device so students could perform operations on large numbers. They still have a place, in today's curriculum in most countries of the world as part of the study of function, but not as a method for calculation as was their initial purpose. Their demise was brought about by the introduction of the arithmetic and then scientific calculators into schools, and so the mathematical content of school courses has adapted as a reaction to changes in available technology. What we must be aware of, however, is how much broader the impact of this new wave of technological change is likely to be on what is now considered core content.

The questions we must consider are what must remain as core concepts and what must now be left behind as artefacts of our progress in the development of mathematical knowledge. These will not be easy decisions as what constitutes the essence of a sound mathematics education is value driven. Doing nothing, though, will mean we teach everything we have done in the past in addition to the new ways of doing mathematics we now have at our disposal. The decreasing amount of time devoted in schools to teaching mathematics will ensure that if this is the option we choose we will most likely do both badly.

The quadratic formula is an example of a particular piece of content that is common place and attracts considerable attention in school mathematics programs. But nearly everything you might expect a student to do with it, with the exception of providing an exact solution, can be achieved in at least three different ways on an entry level graphics calculator. Everything that can be done with the quadratic formula can be achieved with the assistance of a calculator that has a CAS facility. We need to decide if such content still has a place in a technologically rich mathematics classroom or that there is something so fundamentally important about this content to students' learning of mathematics that its place in school programs must be preserved

There is great potential to be explored if we can make some hard decisions in relation to the content that forms the basis of school programs — not the least of which is to provide greater attention to mathematical processes and higher level thinking skills. All authorities responsible for the development of curriculum design in mathematics have attempted to incorporate aspects of mathematical problem solving, modelling and applications, into their programs as a way of addressing the issue of teaching the processes needed to make creative and original use of mathematical knowledge when faced with non-routine or novel problems. The results to date have generally been disappointing. This is in no small part attributable to the attention teachers have been able to give to these areas, given the constraints of time and the availability of facilities powerful enough to explore these matters in a genuine and non-contrived fashion. Graphics calculations can be used as a valuable adjunct to investigative and exploratory approaches to mathematics because they

can allow students to deal with matters, such as extended calculations, methodically and focus on the processes that are invoked when solving a genuine problem.

Assessment

Although a growing body of literature on problems associated with the use of graphics calculators in exams exists, advice available on the design of assessment programs conducted within technologically rich environments is still in a state of development (Jones and McCrae, 1996). This should not be surprising as widespread access is a relatively new phenomenon in itself, both in Australia and elsewhere. It is fair to say, however, that a great deal has been learned, particularly by those who have been associated with processes of implementation in states that have already taken the step of assuming students have access to graphics calculators during high stakes assessment.

What needs to be shared and documented is what we can expect students to do differently in both state-wide high risk testing regimes and in school managed assessment packages, when we assume access to a graphics calculator. This will help inform decisions about how we can best take advantage of the potential for graphics calculators to facilitate finer grained and better quality judgements about what students know and can do mathematically.

We need to understand the ways in which assessment practices need to change so that they are consistent with the context in which students learn. This should be seen as an opportunity for the development of creative assessment tasks and practices and not a matter of adjusting what we already have. The potential of this opportunity will only be realised if the agenda is greater than changing the content of what we teach and assess to accommodate the computational power and the range of facilities now available through these devices. Rather, it is a chance to focus on the processes of doing mathematics and a chance to make judgements about how well students have acquired the ability to make use of these processes. This agenda, at least in stages of schooling that do not involve high stakes assessment, must include a discussion about innovative ways of approaching the assessment of mathematical ideas, concepts and processes, given the power and access to data that students and teachers will have at their finger tips.

Equity

The issue of equity in relation to access is an important consideration and one in which we now have experience with handling through the implementation processes conducted in Western Australian and Victoria. There are other equity related issues, however, which are now likely to be identified given the growing trend toward widespread access.

The first is the issue of gender. What do we know about how girls in our schools interact with and react to this technology? We might have some insights into this area because of parallels that can be drawn with studies on girls and computing but graphics calculators are a much more personal technology and so it is more difficult to make observations about how girls use graphics calculators differently to boys. These differences will be emergent over time and steps must be taken to ensure advice is available on how to best organise

learning experiences so that girls and boys can optimise their potential for success through the use of technology in ways that are best suited to how they learn.

We must also consider the needs of students of either gender for whom technology, at best, offers no advantages and, at worst, inhibits their learning. Have we thought hard enough about those students who become distracted from the learning of a mathematical idea or concept by the learning of a technology based procedure that is intended to facilitate students learning of mathematics? This is again another issue that is likely to become more apparent, over time, in schools and school systems where open access to graphics calculator, or other technologies, is permitted.

These are two matters that are likely to receive a higher profile as issues of access are addressed within schools and systems. There are others that are as yet unknown but will emerge as graphics calculators become more widespread and our experience of using them in the classroom informs our understanding of both the advantages and disadvantages offered by these devices. We must be vigilant and ever ready to address matters related to inequity as they are identified.

Conclusion

In the past, school mathematics has moved forward in response to the influences of curriculum, pedagogy and assessment. We are now seeing glimpses, however, of a school mathematics future, which will continue to be shaped by these influences but will also need to accommodate developments in technology as never before. These are exciting times.

They are exciting times for teachers because technology places in their hands the opportunity to meet students where they are at by providing a broader range of representations for the presentation of new mathematical ideas, thus increasing the chance of connecting with students' already existing conceptual structures. The connections that make up these structures can now be given explicit treatment within a timeframe that that is more consistent with the capacities of a greater number of students. Technology also has the potential to provide teachers with the means to provide the scaffolding needed to accommodate students' gaps in prior learning so that they can still access new ideas and skills in mathematics. Further, mathematically enabled technologies, such as graphics calculators, offer the opportunity to explore pedagogical approaches, such as collaborative learning, in ways that we could not even begin to think about before such technologies became widely accessible. Finally, these technologies open up a whole new range of options, to teachers, in how we assess what student know and can do in mathematics.

More importantly, however, it is an exciting time for students to be learning mathematics, as they now have the tools to shape their own learning and the mathematics that they learn. This shifts their role from passive receivers of a broadcast body of static knowledge to that of a part of a community of inquiry who can genuinely investigate and explore the mathematical landscape that lies before them. Graphics calculators, and other mathematically enabled technologies, place the power to learn in the hands of the learner. Exciting times indeed.

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Assessment Issues Involving Graphics Calculators

Barry Kissane

Australian Institute of Education, Murdoch University, Western Australia

Why the concern for assessment?

Of initial interest is the observation that a substantial portion of the meeting time of this conference has been devoted to issues related to assessment. At first sight, this ought to be surprising. Indeed, my own view is that a graphics calculator is mainly a device to support the learning of mathematics. It has the potential to be important for learning because it can provide access to particular kinds of educational and mathematical experiences (… some of which involve 'graphing' and many of which do not, suggesting that the term 'graphing calculator' is unfortunate, because of its likelihood of reinforcing the misconception that graphics calculators are mainly useful for graphing). Unlike most other technologies used in schools, graphics calculators were *designed* for educational purposes. A graphics calculator is a powerful *learning* device, less powerful as a *teaching* device and not obviously particularly useful as an *assessment* device. From this perspective, the degree of emphasis on assessment would seem at first to be unwarranted.

However, the emphasis on assessment within this conference is a clear reflection of the importance of assessment to thinking about almost any issues in mathematics education. Indeed, the links between teaching, learning and assessment are always recognised officially to be important, reflected in the design of official curriculum documentation at various levels. More than a decade ago, the AAMT's *Discussion Paper on Assessment and Reporting in School Mathematics* (1988) observed the increasing significance of technology for mathematics education, suggesting that there were significant consequences of these changes for assessment:

> *Our conception of what constitutes an adequate mathematical education has undergone considerable recent change, resulting in modifications both to the content and to the teaching methodologies of school mathematics. One such change is an increased emphasis on mathematical processes, as distinct from mathematical content. … The appearance of calculators and micro-computers as tools for both learning and for doing mathematics have changed rapidly and permanently our vision of an adequate mathematical education* (p. 2).

Not surprisingly, the AAMT's *Statement on the Use of Calculators and Computers for Mathematics in Australian Schools* (1996), a revision of the earlier statement of a decade before, reflected the main reason for attending to the links between technology and assessment:

> *Assessment practice should reflect good teaching practice. The use of tech-nological resources as integral aids to learning assumes their inclusion in the assessment process. New approaches to assessment will be required at all levels to better reflect the realities of learning within a technological society* (p. 5).

The significance of assessment in the minds of many students and teachers can hardly be overstated. It has long been clear that what is assessed in mathematics has a good deal to do with how people interpret what is important and valued. The student question, 'Will this be on the test?', or variations of it, is commonly heard at all levels of school mathe-matics education — and no less common at the early undergraduate levels as well.

The nature of and conditions for assessment are widely seen to have significant flow-on effects to classrooms. For example, in those states of Australia in which high stakes external examinations are used, assessment policies regarding the use of graphics calcula-tors have enormous influence over the daily realities of classrooms. Thus, in Western Australia, where senior secondary school examinations are conducted on the assumption that students will have access to graphics calculators, most students have a graphics calcu-lator with them for most of the time in mathematics classes. The calculators that are permitted for use in external examinations are also permitted in classroom tests, school examinations and everyday schoolwork. Of course, the acceptance of graphics calculators for assessment is not the *only* factor here: there has been some re-design of previous courses to accommodate this sort of technology.

In contrast, within states with high stakes external examinations that prohibit the use of graphics calculators (such as New South Wales), graphics calculators are much less evident. It is rather unusual to find students with their own graphics calculator, or even teachers with much active professional interest in this sort of technology. This may be expected to start to change, of course, with the new General Mathematics courses in Years 11 and 12, with talk of graphics calculator use in the relevant HSC examinations at the end of 2001.

As a third example, the state of Queensland is interesting. Without a high stakes external examination, mathematics teachers have more local control over their curriculum. Consequently, there is more diversity of practice evident. In some schools, students have considerable personal access to (and high levels of ownership of) graphics calculators, which have influenced a significant part if the curriculum. In other schools, graphics calcu-lators are not an important part of the curriculum, presumably because of the interests and expertise of the teaching staff at those schools. Thus, while Queensland enjoys some of the advantages of local curriculum ownership, it does not enjoy the advantage of central control.

In each of these three states (and, indeed, in each of the other states and territories in Australia), anyone interested in the role and significance of graphics calculators for math-ematics education is necessarily also obliged to consider the matter of assessment in order to understand present practice and predict likely future directions with any measure of confidence.

What kinds of assessment are relevant?

In many casual conversations about assessment, the relationships between assessment and grading are over-emphasised and the importance of assessment as a means of obtaining and providing feedback is under-emphasised. Indeed, many students (and even teachers and parents) seem to confuse assessment with grading and credentialling issues. Frequently, too, assessment in mathematics focuses on formal assessment, and on testing in particular.

Assessment can and should comprise more than formal assessment, involving testing, for purposes of grading. This is no more or less true in the context of graphics calculators than it is more generally. The AAMT *Discussion Paper* (1988) presented a continuum for assessment, an extract of which is shown below.

Less formal
Less structured

More formal
More structured

←——→

Conversation	Student self-assessment	Classroom checklist	Examinations
	Observation schedule	Projects	
		Assignments	
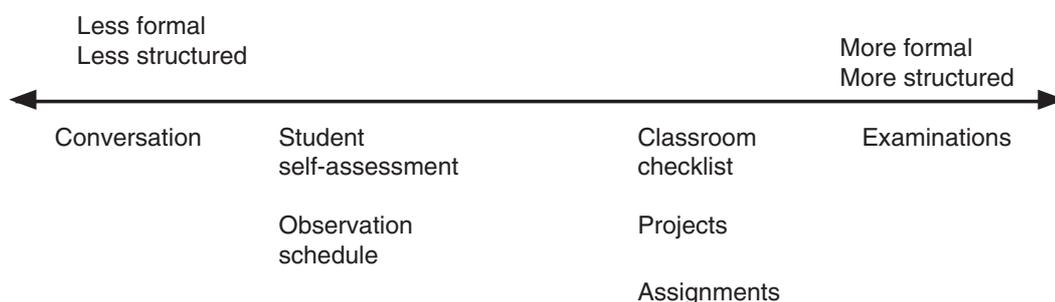
Such a continuum suggests some of the range of assessment practices that need to be considered with graphics calculators in mind. It is salutary to be reminded that, while more formal means of assessment dominate the discussion and thinking of many students and their teachers, less formal methods, such as various observational methods, may in practice provide more useful feedback upon which suitable educational plans can be built. That is, what is most publicly valued is not necessarily what teachers and students will find most useful.

David Clarke (1997) observed that 'teachers are assessing all the time':

> *Historically, assessment has been seen as a separate activity from instruction, and only forms of activity that involved written student product under testlike conditions have been sanctified with the label 'Assessment'. An even more restrictive view of assessment tends to associate assessment only with activities that lead to the grading of student performance. In short, we have tended to label as 'assessment' only a certain subset of our information gathering and exchanging. Is it any wonder that so many students have come to prize earning a grade over genuine learning — or even to confuse the two?*
>
> *A teacher observing a student at work is assessing.*
>
> *A teacher engaging in a class discussion is assessing.*
>
> *A teacher talking to a student about his or her performance is assessing* (p. 20).

In similar vein, the excellent MCTP *Assessment Alternatives in Mathematics* package (Clarke, 1989) highlighted the many forms of assessment tasks: short answer, open-ended, extended-answers, challenging problems, investigations and investigative projects.

In principle, then, assessment associated with graphics calculators can be oral or written, external or internal to the school and more or less formal. However, formal assessment often carries more weight for grading, is often regarded as more credible, is usually undertaken under controlled conditions such as timing, facilities and communication with other people. Although such assessment is possibly more reliable, since the conditions can be readily replicated, it is probably a less valid way of obtaining important information about student learning than is less formal assessment. Given its public prominence, however, it is necessary to consider formal assessment in detail, with the relevance of graphics calculators in mind.

Issues in formal assessment

Despite much good professional advice, tests and examinations continue to be very important in the Australian educational culture, both within classrooms and external to classrooms. As noted above, the flow-on effects of these are most obvious, since school assessment is usually strongly influenced by external assessment, especially when issues of university entrance are involved (a high stakes situation). However, the flow-on effects of the ways in which graphics calculators are used in formal assessment with a more low stakes character (such as classroom tests) are also likely to be considerable. Hence, this section of the paper focuses on issues for formal assessment.

Many of these issues have been previously described and elaborated in some key papers written by Jen Bradley, Marian Kemp and myself, reflecting philosophical, practical and analytic perspectives. Interested readers are referred to the details of these. The papers were written in the context of the *integrated* use of graphics calculators in mathematics education, and derived from our experiences at Murdoch University and elsewhere.

Levels of calculator use

A key issue concerns whether or not graphics calculators are used in formal assessment at all, discussed at length by Kissane, Bradley & Kemp (1994). There is a continuum of possibilities, all of which are used at present in parts of Australia:

- Unrestricted use of *any* calculators
- Unrestricted use of a restricted set of calculators
- Calculator-neutral assessment
- Calculator-free assessment (for some assessment components)
- No calculator use permitted

Another way of thinking about this is to identify three possible choices to be made for calculator use in examinations:

- *Required*

  Access and instruction are assumed.

  Students are expected to use where suitable.

  Student choice is to be exercised — and assessed.

- *Allowed*

  (Perhaps begrudgingly?) Students can use calculators if they want to do so.

  It is assumed that some students will not have access.

- *Disallowed*

  Graphics calculator use is forbidden.

  This category has a strong suggestion of control (and gate-keeping?) associated with it.

These issues are discussed in some detail in Kissane, Bradley & Kemp (1994), so that it is not appropriate to repeat the arguments here. Rather, it ought be noted that the arguments for calculator-neutral testing are difficult to make convincingly, in part because it is hard to comprehend that a calculator-neutral test is actually possible without significant sacrifice of content validity, especially given the nature of most senior secondary mathematics courses in Australia today. Both calculator-neutral and calculator-free testing come perilously close to sacrificing the earlier principles suggested of ensuring a measure of coherence between teaching, learning and assessment.

A typology

Based on an analysis of practice, Kemp, Kissane & Bradley (1996) suggested a typology of student uses of graphics calculators in examinations. Three possibilities are identified:

- calculator use is expected

- calculator is used by some students, but not by others

- calculator use is not expected.

Although first constructed some years ago, this typology still seems to be of value today, both at external examination level and within courses; our experience extends to both of these settings.

There are three possible situations in which graphics calculators are expected to be used in examinations:

- students are explicitly advised or even told to use graphics calculators;

- alternatives to graphics calculator use are very inefficient;

- graphics calculators are used as scientific calculators only.

To illustrate but one of these cases, consider an examination task requiring students to solve the equation, $x^3 = x + 4$. A modern graphics calculator can give all solutions quickly (and thus efficiently), as the two screen dumps from a Casio cfx-9850G calculator below suggest:
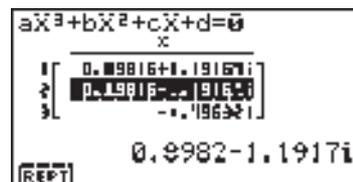
Analytic and graphical solutions of such an equation are slow and less comprehensive, and thus much less efficient. In contrast, the calculator provides both real and complex solutions, almost immediately after the coefficients are entered.

To illustrate situations for which graphics calculators might be expected to be used by some students and not by others, consider the task of investigating the effect of an annual compound interest rate of 6%, starting with an initial amount of \$14 000. For the related task, a student may choose to use their calculator as shown below:

```
                        14000.00
Ans×1.06
                        14840.00
                        15730.40
                        16674.22
                        17674.68
                        18735.16
```

Another student may prefer to use a suitable compound interest formula (involving an exponential function). The choice may depend on the level of comfort a student enjoys both with the calculator and with the relevant mathematics. It may even be that somebody quite comfortable with *both* the relevant mathematics and with the calculator may choose one of these on some occasions and the other at other times. For instance, the calculator solution would be tedious and error-prone if the amount required involved a long time (say twenty years) into the future. On the other hand, a formula here would provide information about the final value but no information about annual changes.)

There are several circumstances in which we would expect calculators not to be used in assessment situations:

- exact answers are required;

- symbolic answers are required;

- written explanations of reasoning are required;

- task involves extracting the mathematics from a situation or representing a situation mathematically;

- graphics calculator use is inefficient;

- task requires that a representation of a graphics calculator screen will be interpreted.

To illustrate one of these cases, students might be asked to interpret the Texas Instruments TI-82 calculator screen below, to find a value of x for which f(x) has an approximate value of 6.

```
   X      Y1
  0      0
  1      1.1052
  2      2.4428
  3      4.0496
  4      5.9673
  5      8.2436
  6      10.933
Y1◼Xe^0.1X
```

It is worth noting that there are sometimes difficulties in classifying items with such a typology. For example, a task may be written to apparently demand that analytic methods be used (in preference to a calculator), when in fact a competent student may use their graphics calculator to thwart this intention. For example, consider the task:

Solve exactly $e^{x^2-5} = 1$

The use of the word 'exactly' suggests that a numerical method is inappropriate. In this case, the numerical result of $\sqrt{5}$ is obtained on a TI-82, using a *solve* command, and an astute student can square this result to see that it must be the square root of 5:

```
solve(e^(X²-5)-1
,X,0)
          2.236067977
Ans²
                    5
```
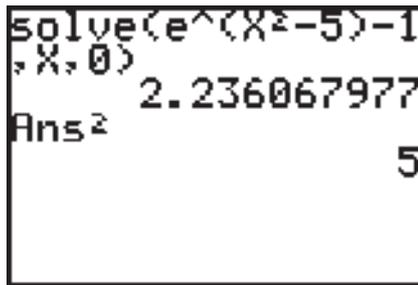
The construction of examination tasks is clearly important, both to communicate intended student behaviours and also to ensure that the performance expected is being prompted. To illustrate this point, consider the following theme and variations on an examination task discussed by Kissane, Kemp & Bradley (1996):

Solve $(x + 2)^2 = 9$

Solve $(x + 2)^2 = 8$

Solve exactly $(x + 2)^2 = 8$

Solve $(x + a)^2 = 9$

Solve $(x + 2)^2 = 9$ and explain why there are only two solutions.

These examples suggest that tests and examinations in which students have graphics calculators need to be written carefully — as indeed do all formal assessment tasks, of course. The relationship between a test and the use of graphics calculators ought be a conscious *design*, not just occur accidentally, as suggested by Kemp, Kissane & Bradley (1996). As well as helping analyse and understand student responses to examination tasks, this typology may reveal our intentions or habits. Bradley (1999) provides several examples of graphics calculator use in a recent high stakes examination.

What is to be recorded?

Another issue concerns the fact that it is usually hard for students to write down adequate steps to 'show their working' on questions for which they have made significant use of a graphics calculator. Usually, a listing of calculator keystrokes is not particularly helpful (although it can be quite illuminating provided the person reading it is thoroughly familiar with the particular calculator used.)

Kissane, Bradley and Kemp (1994) have suggested two general principles for this issue and discussed them briefly:

> *If we require working to be shown, it should be worth showing in its own right, and not only as a means of awarding partial credit.*
>
> *If only part marks are to be awarded for numerically correct answers, for which working is not provided, then this should be stated explicitly in advance.*

Equity in assessment

Discussions about the place of graphics calculators in assessment rarely progress far without some mention of issues of 'equity'. In some states, the widespread (and official) unease about 'equity' issues is largely responsible for a delayed introduction of graphics calculators into schools. In other states, a concern for 'equity' has strongly influenced assessment practice, especially in high stakes external examinations. Several issues associated with equity have been canvassed in Kissane, Bradley and Kemp (1994).

A key issue is whether or not it is equitable to *not* allow graphics calculators in external examinations. As Kissane, Bradley & Kemp (1996) argue, one consequence of preventing students from using graphics calculators in examinations is that students in the least-resourced schools are consequently denied access to this useful technology. There is rarely money for 'frills' in such schools, but resources will be found for 'necessities'. When an examination is prepared on the assumption that students have access to graphics calculators, and the corresponding course has been designed accordingly, decisions can be made to allocate funds suitably at the school level. It is much harder to argue for appropriate resourcing at the school level for a technology that is merely 'recommended'. Around Australia, school budgets frequently contain substantial provisions for Information Technology resources, usually in the form of computer laboratories and associated telecommunications networks. A small shift of some of these funds towards the much less expensive technology of graphics calculators is possible, when graphics calculators are seen as necessities rather than as luxuries. These issues are clearly less of a concern for schools that are well resourced, and are well located within relatively affluent communities.

There are many different models of calculators, and so attention is needed to ensure that students are not unduly advantaged or disadvantaged for assessment purposes with respect to their peers by their particular choice of calculator (which may not have been their own choice, of course). Prudence suggests that both schools and individuals may need some careful guidance to ensure that underpowered calculators are not purchased unknowingly and also that students are not unduly advantaged over others because of especially powerful calculators (which usually cost more to purchase, of course). Suitable specifications are needed to achieve these two ends, as described briefly in the next section.

All graphics calculators are programmable, and it seems sensible to allow the programmability to be used to upgrade calculator capabilities where they are comparatively modest. As noted by Kissane, Bradley & Kemp (1994), this strategy was originally used in the Advanced Placement examinations of the College Board in the USA, where minimum calculator specifications were demanded of calculators, and programs used to add these in the case of less sophisticated calculators. Of course, such a strategy is of no value if calculator memories are required to be cleared before an examination commences, which is why such a practice is most undesirable.

Calculator memories that allow programs to be added, thus potentially reducing some differences between models, also allow text to be entered, in some cases in an awkward way. One response to this possibility is to demand that memories be cleared before examinations begin, but this seems to eliminate any possibility of calculator upgrading through the use of programs. An alternative strategy, employed in some places to apparently good effect, is to allow students to take some pages of notes with them into an examination. Such a practice raises some interesting issues regarding the nature of mathematical activity; for example, how critical is human *memory* in mathematics?

The use of 'calculator-neutral' testing to protect equity seems particularly problematic. In the first place, it is not clear that it is actually possible to construct (valid) calculator neutral tests. In the second place, the use of such an approach sends an odd signal to the community: that graphics calculators are 'useful' enough to encourage their use and purpose, but not useful enough to be of any advantage in an examination. It is very difficult to make such a message convincing.

It needs to be acknowledged that students who have regular and personal access to a graphics calculator (e.g. one that they personally own or that is theirs on long term hire or loan from a school) would seem more likely to make good use of it and to become familiar enough with it and use it well, than are those students whose physical access is restricted to the use of a class set of calculators handed out by the decision of the teacher and collected up again at the end of a lesson.

In recent years, it has become evident that equity issues are not restricted to access to the physical technology, however, but also concern access to quality guidance and support. There are at least three aspects of this: students need to have teachers who are competent users of graphics calculators themselves, so that they may get suitable instruction in their appropriate use; they need school mathematics courses to be taught in a way that integrates technology into them, rather than regarding it as an occasional add-on; they need curriculum materials which integrate calculators into the mathematics. These three aspects of equity are quite difficult to deal with. None is easy to bring about, and none is evident at first, since attention often is focussed on the provision of hardware.

Calculator specifications

As noted in the previous section, for equity reasons, the range of allowable calculators in formal assessment settings is often restricted. A common example of this is that calculator keyboards with a QWERTY keyboard are banned from use in examinations (although of course that does not mean that they could not be used at a local level within a classroom or even within an assessment context where all students had comparable access to them). The banning of calculators with QWERTY keyboards (originally by the Advanced Placement examination authorities in the USA) clearly reflected a concern for easy text entering and recall. However, such an indirect specification seems to be a stop-gap measure, since it does not address precisely the concern at hand: the power of the technology and the possibility of students having differential access to it. Indeed, it now seems a little ironic that this specification, which prevents the Texas Instruments TI-92 calculator from being used in the relevant examination, does not prevent the use of the companion TI-89 calculator, which is more powerful in terms of capabilities — but lacking a QWERTY keyboard. (Readers interested in regulations regarding calculator specifications will find some links to them on the web site: http://www.staff.murdoch.edu.au/~kissane by

following the links to graphics calculators.)

A more prudent approach to dealing with calculator specifications may be more direct, such as not permitting calculators with '*Extensive* symbolic manipulation' to be used in high stakes examinations. While this still requires careful definition of both 'extensive' and 'symbolic', usually by reference to particular calculator models, it seems more sensible than hoping other attributes (such as a QWERTY keyboard) will carry the information suitably. Of course, all calculators provide a form of symbolic manipulation; after all the numerals are symbols for their corresponding numbers. Assessment regulations that bar *any* symbolic manipulation are unlikely to help.

An emerging issue for calculator specifications concerns 'flash memory', now available on a number of calculators. The capabilities of calculators with flash memory can be upgraded as new features become available (or are seen to be popular, or even desirable). While this appears on the one hand to be a good strategy for calculator design, it creates a new problem for high stakes assessment, since the calculator capabilities cannot be determined without actually operating it. While this is also true to an extent of any programmable calculator, at least it is fairly well understood what kinds of programs are available (and perhaps more importantly, what kinds are *not* available). A decision on suitability can be made by examination authorities. Typically, programs for calculators do not add significantly to the basic inbuilt capabilities of a calculator, but rather make use of them in tailored ways. It is much harder to deal with this issue when it is the capabilities themselves that are being upgraded. As for programming, flash memory upgrades are not restricted to those provided by calculator manufacturers, so that reliable informed advice on the possibilities is especially difficult to obtain.

The last few years have seen the development of algebraic calculators, which have extensive symbolic manipulation capabilities built in to them, of particular relevance to algebra and the calculus (see Kissane (1996; 1999) for some examples of these). The screen below shows a few algebra examples from the Texas Instruments TI-92.
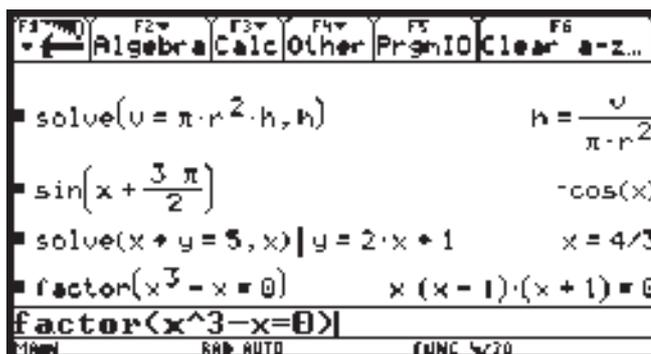
The next diagram show a succession of screens generated by the Casio Algebra fx 2.0.
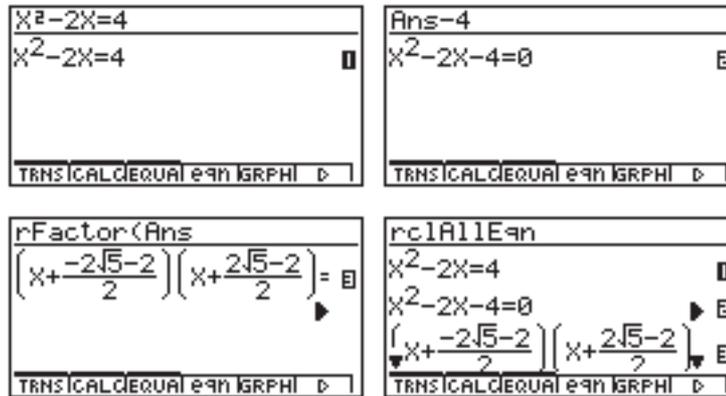
The assessment issue related to such calculators is that it is much harder to distinguish student thinking from machine work and thus to interpret what is written down in a formal assessment setting. Algebraic calculators are significantly more expensive (today) than other kinds of graphics calculators, and so there are more likely to be equity issues associated with their use. Without doubt, dealing with the assessment issues associated with algebraic calculators is one of the biggest challenges facing curriculum and assessment authorities in the near future. Such problems are less acute at the local level (e.g. at a single school), where it is much easier to observe, control and understand the access and equity issues.

Electronic algorithms

Another assessment issue is that some calculator programs seem to be designed mainly to allow exam questions to be answered. These are often fairly large programs, but do not require students to key them in, since electronic transfer of programs is possible with most calculator models these days. The screens below show a calculator program of this kind for handling routine questions concerned with geometric progressions.
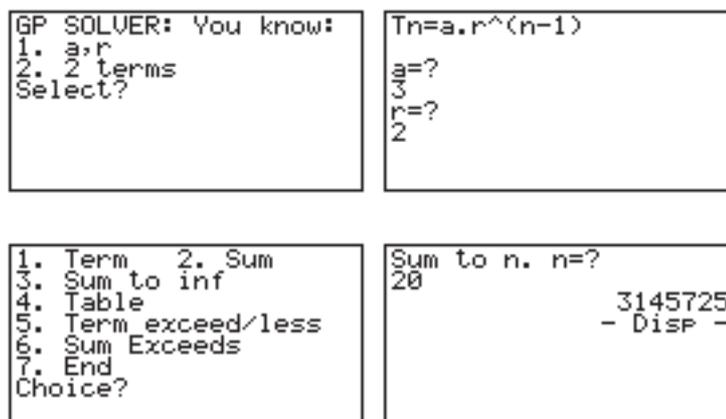
Such programs are conceptually similar to routine algorithms, requiring only that students recognise key aspects and perform a routine computational procedure to obtain the desired answer. It does not seem as if much useful assessment information can be gained from student responses to routine questions that can be dealt with in this kind of way. (Of course, it is arguable that the same is true of paper and pencil algorithms of the same kind.)

The most obvious way of dealing with such possibilities is to not ask routine questions of a kind that can be answered numerically like this. Another is to demand that students 'explain' their working in some sense (although this is not always easy to ask — or to do). It is important to note too that there is a substantial intellectual difference between students writing such programs for themselves (which will usually reflect significant learning) and using programs written by others (which often will not reflect much learning of mathematics at all).

Assessing calculator usage

Mathematics assessment is concerned with how well students know, understand and are able to do mathematics. But in a context in which technology is important, we may also wish to assess how well students can use a graphics calculator (without misunderstanding this as mathematics). It is important for students to make efficient use of technology, although it is probably easier to assess this informally than formally. As already noted, it is quite hard for students to 'show their work' when using a calculator; however, it is relatively easier to watch students using calculators and appraise the efficiency and sensibleness with which they do so. Assessment of calculator use seems to be important, if teachers are to provide useful feedback to students to improve their performance in this respect.

In summary

Although many of the issues raised here involve formal testing, assessment ought to involve (much) more than testing. Some coherence between assessment practices and the mathematics curriculum is critical. Graphics calculator use in assessment is desirable and even necessary in the quest for this coherence, but will demand some rethinking of assessment practice. Although it has always been important for tests to be carefully designed, it seems that the possibility of calculator use in tests makes this even more critical.

Finally, two observations about assessment, made almost a generation apart, offer some food for thought when considering the many assessment issues related to graphics calculators. The first serves to remind us of the considerable limitations of formal assessment:

> *Contemporary assessment recognises the inadequacy of the 'Assessment as measurement' metaphor. Our goal is now 'Assessment as portrayal'* (Clarke, 1997, p. 65).

The second reminds us of the significance of assessment in the context of teaching and learning mathematics:

> *Nobody ever got taller just by being measured* (Cockcroft Report, 1980).

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Note that several of the above publications are available from the following web site: http://wwwstaff.murdoch.edu.au/~kissane/epublications.htm

Acknowledgement

Major Presentations

Graphical Calculators in Mathematical Courses: Some Assessment Issues

Colin Croft

Hewlett-Packard Australian Calculator Operation, Western Australia

Introduction

The process of the introduction of graphical calculators into mathematical teaching has necessarily affected assessment. In particular I look at the transition phase from pre to post graphical calculator usage, examining their effects on questions in tests, examinations and investigative/discovery assignments.

In this Master Class I will be reviewing some of the conclusions I have come to as a classroom teacher. More importantly, in view of the still changing nature of the topic, I will be putting some topics to the group for discussion. Amongst these topics are some which I have found to cause concern with classroom teachers.

What effect has the change had?

In my personal experience, the effect has been both larger and smaller than might first be expected.

Smaller

In a review of five years of past examination papers (Applicable Mathematics course in Western Australia) which I worked on for the Curriculum Council in 1996, the large majority of questions were found to require no modification at all. Only about one in ten were trivialised to the point of being discarded, with the remainder able to be modified to embrace the new technology.

My experience in writing examination and test papers as a school teacher has backed up these conclusions. With a little imagination, it has not been difficult to write questions which still test the concepts and skills required, while also using the capabilities offered by the graphical calculator.

Larger

Graphical calculators have provided a welcome change to the setting of questions in that they can become less sanitised and closer to 'real-life'. Losing the requirement that answers be 'nice' makes the questions more realistic and also easier to write.

An additional advantage is that the speed of the calculator allows the inclusion into exam-

inations and tests of questions which are to some degree exploration or discovery style. This is something which has always been desired but difficult to achieve. As will be seen later, there is a need to be careful with this but it can be used to great effect.

In addition, it is possible to set investigative or discovery style assignments, open-ended or semi-closed, which are both challenging and rewarding for the student.

Updating questions

When looking at the change in style of question from pre-graphical calculator to post, assessment questions seem to fall roughly into categories, broadly outlined as:

- questions which, by their nature, are unaffected by the use of a graphical calculator;

- questions which are made redundant or trivial by the use of a graphical calculator. Most of these may be useable if re-written but there are issues which must be considered in this rewriting:
    - the thrust of the question may change in the revision process; and
    - the degree of difficulty of the question may change;

- questions which were not feasible before the availability of a graphical calculator but may now be posed.

The final category is the one which is potentially the most interesting and rewarding. Teachers who have worked imaginatively with assessment and graphical calculators find that they can now set questions which are wider in scope, more realistic and which encourage the student to discover mathematical concepts and principles for themselves.

In discussing discovery style questions, I should mention that I am not confining myself here only to those questions used in examinations and tests. As always, the major constraint with this style of question is with the time requirement and a common way to deal with this is to introduce the question using a worksheet which the students may have a week to work on. A validation test may then be used to ensure that it is the student's understanding which is being tested rather than that of a friend, sibling or tutor.

The question of how 'open-ended' to make these investigations is a matter for decision by the teacher according to his or her personal preferences and the demands of the educational system within which the assessment is set. Clearly it is easier to assign marks to an investigation which is heavily guided. Many teachers have, on the other hand, found out how interesting it can be to let the student 'follow their nose'.

Testing of low level skills

One of the issues which is often raised, particularly by curriculum boards considering whether to allow graphical calculators into the education system at all, is the issue of how the power of graphical calculators is perceived to cause problems in testing 'easy' concepts. A simple example of this is in testing the process of differentiation. When many graphical calculators have the capability to perform algebraic differentiation it can become harder to

ensure that it is the student's ability which is being tested, and not the technological progress of calculators.

An important point to remember in considering this issue is the difference between the tool and the use to which it is put. To bring things down to the most basic level, consider the basic skill of being able to drive a nail with a hammer, and the use to which it might be put: making a table. If a nail gun is invented which makes the hammering skill of less value, should we force our students to continue to use the hammer and neglect the new technology, or should we place the emphasis squarely where it belongs: on the end product of the table?

Two of the possible responses to this problem which I see commonly taken are listed below. I would value contribution from the group of others:

- While it is still important that the student demonstrate an understanding of the skill, one might consider the question that if the concept is so mechanical that a calculator can do it, how much weight should be attached to it in testing? Devaluing this component of the question can remove the problem's immediacy.

- Alternative methods of testing are always a possibility. An example in the case of differentiation might be to conduct a small, ten minute, spot test of this skill alone and to disallow the use of the calculator for that test. Not even the most avid enthusiast would claim that a graphical calculator must be used in every single assessment item. This has always seemed to me, however, to be the coward's way out.

Some examples

The pages which follow contain some examples taken from tests and examination papers used in my school. In each case there is some discussion following.

A. Questions which are unaffected by the use of a graphical calculator:

> Let *OABC* be a rhombus, with OA = **a** and *OC* = **c**. Draw a diagram, find an expression for the dot product of the diagonals, and hence show that the diagonals of a rhombus intersect at right angles.

In the question above it is the subject matter which makes it unsuitable for solution using a graphical calculator. There will probably always be questions like this.

Please note that it should be remembered that the pace of technological change is such that this may not remain true indefinitely for geometrical questions. Applications already available on some top end calculators show that geometrical proofs may well enter the standard capabilities of all graphical calculators in the near future.

> Given that the line $x + 3y = -2$ is perpendicular to the function
> $f(x) = ax^3 - 3x$ at $x = 1$, find the value of a.

This example is an interesting one in that it requires the student to demonstrate an understanding of a number of mathematical concepts, while still allowing the use of a graphical calculator in many parts of the process of finding a solution. The only problem with this question is that its level of difficulty is fairly high.

B. Questions which are trivialised but may be retrieved by being re-written

The original...

Find the area enclosed by the curves $f(x) = x^2 + 2x - 3$ and $g(x) = 3x + 3$.

This could be rewritten as:

Find the area enclosed by the curves $f(x) = x^2 + 2x - 3$ and $g(x) = 3x + 3$.

Note: It is expected that the calculator will be used for this question and hence full working is not required. HOWEVER showing some working, such as a quick sketch with intersections marked, might help us to allocate part marks if you go wrong!

The addition used above may seem to be trivial but this is, I think, an over reaction to what may be a perfectly acceptable adaptation if not used exclusively. The problem of ensuring that students using graphical calculators show a sufficient working trail which will allow part marks to be allocated is not at all trivial and causes concern to many teachers during the transition phase.

My experience is that discussion of this need with students, together with consideration of possible strategies and feedback to them on their degree of success will largely alleviate the problem. However, one must watch for the student who decides that their only recourse is to do the entire problem by hand and not to use the calculator at all!

Finally, as has been pointed out by others, the desire of the teacher to assign part marks for working is not, in and of itself, sufficient reason for requiring that working.

Another possibility (below) is to split the question up into parts.

(i)   Draw a neat, labelled sketch graph of the area enclosed by the curves $f(x) = x^2 + 2x - 3$ and $g(x) = 3x + 3$.

(ii)   Use your calculator to find the intersections of $f(x)$ and $g(x)$ and mark them on the sketch.

(iii)   Find the area between the two curves.

(iv)   Use your sketch to decide whether the area enclosed by $f(x) = x^2 + 2x - 3$ and $g(x) = 3x + 1$ will be larger or smaller than the area found in part (iii).

(v)   On your sketch, draw a line which has the same gradient but which encloses an area of size zero.

(vi)   Find the equation of this line.

This question now uses the power of the graphical calculator, while still testing the student's conceptual understanding of the area between curves. As mentioned earlier, it is important to try to ensure that the degree of difficulty of the question does not increase too markedly. In this case, parts (iv) and (v) allow students with some degree of conceptual understanding but without the ability to solve a question like (vi) to achieve a degree of success. The weighting chosen for part (vi) can also influence this.

Questions can obviously also be created so that a graphical calculator offers no real advantage.

> The graph of a cubic equation with integer roots is shown right.
> Find its equation in factor form.
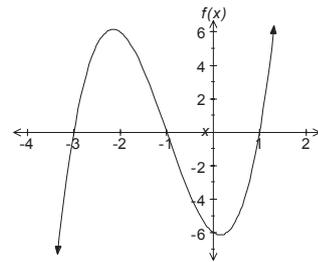
As mentioned earlier, there are issues which must be considered when rewriting a question.

Firstly, the thrust of the question may change in the revision process. One must be careful that if the concept or skill being examined changes then the teacher is aware of this.

Secondly, the degree of difficulty of the question may change. For example, a common method used by teachers to re-write a question is to insert letters for parameters in equations. Although this is effective it also increases the level of difficulty. There are usually other ways to achieve re-writes that are just as effective.

A challenge for readers

What would you do with the following questions? How could they be changed, either by re-wording, rewriting or replacing, so that they become good questions again?

1. Find the roots and turning point of the quadratic $y = x^2 - 3x - 1$.

2. If $A = \begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix}$ then find:

    (i) $A \times B$

    (ii) X such that $AX = I$

3. Solve the simultaneous equations: $\begin{cases} x + 3y = 7 \\ 2x - y = 0 \end{cases}$

4. The weights of newborn babies are assumed to be normally distributed with a mean of 3.5 kg and a standard deviation of 0.5 kg.

    What is the probability that a randomly chosen newborn baby will have a weight which is:

    (i)   between 3 kg and 4.5 kg.

    (ii) exactly 3.5 kg.

Among many possible changes, some good and some not, are:

1. By completing the square, find the turning point of the quadratic $y = x^2 - 4x - 5$.

or… Factorise $y = x^2 - 4x - 5$ and hence find the x intercepts of the equation.

or… Draw a neat labelled sketch graph, showing the roots and the turning point.

2.   If $A = \begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix}$ then find:

(i) $A \times B$

(ii) X such that $AX = I$

(iii)   For what value(s) of k does the matrix X not exist?

3.   Using any algebraic method and showing working, solve the simultaneous equations
$$\begin{cases} x + 3y = 7 \\ 2x - y = 0 \end{cases}$$

or…   Re-arrange the simultaneous equations $\begin{cases} x + 3y = 7 \\ 2x - y = 0 \end{cases}$ into the form $y=mx+c$.

or…   Graph the equations on your calculator and hence find the solution to the simultaneous equations.

4.   The weights of newborn babies are assumed to be normally distributed with a mean of 3.5 kg and a standard deviation of 0.5 kg.

(i)   Sketch a normal curve and on it mark the area which would be used to find the probability that a randomly chosen newborn baby will have a weight which is between 3 kg and 4.5 kg.

(ii)   What is the probability of a randomly chosen baby weighing exactly 3.5 kg.

(iii)   In the real world, digital machines used to weigh babies will round off to the nearest tenth of a kilogram. Explain how this might influence the answer to (b) and what a more accurate 'real' answer might be.

Questions not feasible before, but which may now be posed

Discovery style questions can be very interesting and rewarding, both for the teacher and for the student. Clearly there is a difference between posing this type of question in an examination or test and posing it as an assignment. A common method is to use an assignment on which the student can work, often for up to a week or even more. If required, a validation test can be used to ensure that the final assessment result obtained for an assignment is that of the student rather than the tutor, parent or sibling. Different educational systems in Australia have treated this point in different ways. For example, Queensland's system allows teachers to encourage long term open-ended investigative work.

Discovery-style questions, given the already stressful environment of a test or examination, may be a little threatening for mid to lower ability students. However, this will be true regardless of whether a graphical calculator is used, and the use of a graphical calculator will help these students by alleviating some of the more mundane requirements of the question and allowing them to concentrate on the underlying concept.

One of the possible ways to help with this problem is to lead into the discovery question gently (e.g. with plenty of hints) without expecting a jump in understanding which is too large. For example, a model might be: 'normal' question, 'normal' question, 'normal' question, 'small discovery' question, etc.

The test question below takes the path of asking the student to use the calculator's capabilities to explore the idea of domain. This was used with a group of students who had not formally been introduced to the concept before. Their only exposure had been in being told, 'You can't square root a negative'. In the event, the hint given in part (a) was perhaps a little too obvious.

Jane was graphing the function $f(x) = \sqrt{x+2}$ on her calculator and obtained the graph shown below.
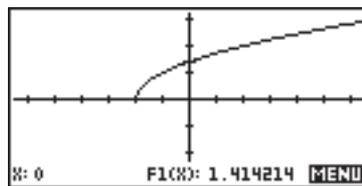
(a) Jane noticed that the graph did not extend past -2 on the left. She knew that the square root function normally only worked on positive numbers (and zero). Explain why Jane's graph goes down to -2.

(b) Give equations of graphs which would:
   (i) extend down to -5;
   (ii) extend from positive 3 onward.

An example of an extended graphical calculator investigation, including the validation test, can be found at http://members.iinet.net.au/~ccroft/example.pdf.

In this example paper, which was given to Year 11 Introductory Calculus students (early in the course), students had a week to consider the assignment before being required to take the validation test. This example is guided and is relatively closed in its expectations in order to simplify the marking process. Students were able to ask questions of the teaching staff but teachers were asked to respond with hints only.

Conclusion

The area of assessment raises many issues in relation to the use of graphics calculators. This paper has illustrated some practical possibilities which may prove useful in helping colleagues find their way forward.

Summary and Reflections

John O'Brien

Board of Senior Secondary School Studies, Queensland

In an interactive session Colin presented three aspects of myths and opportunities:

- How much of an effect will/do GCs have?

- How can we manage the change?

- What is the next generation of challenges?

How much affect?

Colin has examined items on past papers and found that:

- in one sense the effect will be/is small

  — 65 per cent of items remain valid even when students have access to GCs

  — only 10 per cent of the items were trivialised

- yet, in another sense the affect will be/is large

  — GCs offer students opportunities and freedom to explore real-world situations, investigate mathematical models and discover new knowledge.

Managing the change

Colin proposed that managing the change in assessment conditions where students have access to GCs involves keeping many items, losing some items, changing some items and creating new items.

- *Keeping items*

  Examples of items that remain valid under the new conditions include: showing that the diagonals of a rhombus intersect at right angles; finding the exact root of equations with irrational roots. Participants were asked to contribute other items or types of items to the discussion. There were plenty of ideas.

- *Changing items*

  Items or assessment strategies that could remain valid with some modification. Examples included: instructing students to show working; break items into parts. There was some discussion on the matter of identifying the important techniques which, although able to be performed using GCs, were considered to be necessary (at present, at least) pen-and-paper skills. It was suggested by some, but not agreed,

that calculator-free conditions may be necessary.

- *Creating new items*

  Items that encourage students to discover new knowledge and investigate situations from a mathematical perspective (such as, exploring the domain of functions) were agreed by many to be rewarding and motivating for students (and teachers) and to be an important benefit of GCs. Colin warned that such items are threatening under strict exam conditions and most agreed that items of this type are best set to be completed over time and involving others. Colin handed out some items which, in his words, 'GCs have wrecked' and asked the participants to work in groups to propose ways to recover the validity of the items.

The next generation of challenge

The challenges in the future involve accommodating:

- the variable levels of item difficulty in different solutions to open-ended items

- the different levels of assistance provided to students

- the range and balance of assessment conditions.

In closing it was generally agreed by all that *every good learning experience is a good assessment opportunity*. We should make the most of them.

It Is a Poor Tradesperson Who Blames Their Tool

Anthony Harradine

Prince Alfred College, South Australia

For a long time some experts have blamed the perceived erosion of our youth's mathematical skills on the advent of the calculator. I am too young to know if a similar cry was made when the slide rule was introduced. The calculator has been seen as the Devil's tool in many people's eyes.

The famous Triton workbench, when in the hands of a master user, is a wonderful tool. It allows the user to 'learn' about their craft in the true sense of the word. In the hands of the uninitiated it is next to useless — much the same as a graphics calculator.

The Dimensions of Learning (DOL) program is a well regarded synthesis of what should occur during the process of education to ensure the learning that occurs is well rounded and will result in a well formed mind. Traditionally, mathematics classes exposed students to only one of the four academic dimensions of this program, namely Acquiring Knowledge. Dimensions like Extending and Refining Knowledge and Productive Habits of Mind have been given little air time. Often the acquisition of knowledge has occurred in less than desirable ways.

This presentation looks at practical ways in which electronic technology, like the graphics calculator, can bring all the dimensions of learning into the mathematics classroom and raise questions about the value and cost of such progress.

The practice

What is a Stenduser?

Initially the use of such a beast is designed to develop positive attitudes and perceptions to the learning of mathematics. *Stendusers* (a term used at PAC for an activity that *st*arts a topic, *end*s a topic and hopefully enth*use*s the students along the way) often require the use of some form of electronic technology. They are meant to be simple enough for the students to get a start on, hard enough so that they can not solve them without learning some new theory, and interesting. They may be worked on throughout the topic of study and can be recalled at any time to help pursue new knowledge. The final solution of a Stenduser normally requires students to work in the higher dimensions of the DOL framework.

The Stenduser that follows is aimed at getting into the algebra beyond Year 8. Students will have only very basic algebraic skills (the distributive law, $a(b + c) = ab + ac$ being the most complex rule to which they would have been exposed) when interacting with this activity.

Work your way through the following Stenduser.

Stenduser — an example

Year 9

The difference between any pair of consecutive square numbers

1. Choose seven pairs of consecutive square numbers and enter them in the first column of the table below. Then complete the rest of the table.

A pair of consecutive square numbers	The difference of the pair
25, 36	11
3721, 3844	123

2. Study the difference column, what appears to be a consistent feature of the numbers?

3. Use a graphic calculator or spreadsheet to calculate the difference between many pairs of consecutive squares. Does the feature in the table hold for the many specific cases you have tried?

4. Form a conjecture, in English, (which must be about the difference of *all* pairs of consecutive odds) that summaries your observations.

5. Use a pronumeral to help you write down every square number.

6. Now write down every pair of consecutive square numbers.

7. Use a pronumeral to write down every odd number.

8. Evaluate many specific cases of $(x+1)^2 - x^2$ and $2x+1$ in *one table* on the graphic calculator or a spreadsheet. What do you notice?

9. Now re-write your conjecture from question 4 but replace the English with an algebraic equation.

10. Prove that your conjecture is true.

EXTENSION

1. Evaluate many specific cases of $(x+5)^2 - (x+4)^2$ and $2x+1$ in one table on the graphic calculator or a spreadsheet. What do you notice?

2. What does $(x+5)^2 - (x+4)^2$ represent?

3. Is $(x+5)^2 - (x+4)^2$ always odd? Then why is $(x+5)^2 - (x+4)^2$ not equal to $2x+1$?

4. Discover an equivalent form to $(x+5)^2 - (x+4)^2$.

5. Show that the equivalent form you have discovered is always odd.

What is a Learning Journey?

To learn we must interact. These days, students sitting and listening passively should be long gone. They are, however, not. Not enough materials are readily available that offer students learning journeys — text-books are not big on such activities. A Learning Journey may introduce students to a new piece of knowledge in an inductive or deductive manner, or take them on a journey of extending and refining knowledge they already have. The example below is an extending and refining task — the rarer of the two breeds.

LJs are my present passion. I believe they are the key to changing the way our students think about mathematics. To be successful though, students must start on a diet of this type of approach to learning very early or they will be too lazy to succeed in such tasks. It is always easier to be told, but much less rich than constructing the learning for yourself.

Now work your way through this activity. It is a Learning Journey.

Learning Journey — an example

Year 11

The zeros of a quadratic family

Have you ever considered how certain *families* of quadratic functions behave with respect to their zeros? Do they behave in a predictable manner?

Consider the family of which the following *specific* quadratic functions are members:

$$f(x) = -3x^2 - 3$$

$$f(x) = -2x^2 - 2$$

$$f(x) = -1x^2 - 1$$

$$f(x) = 1x^2 + 1$$

$$f(x) = 2x^2 + 2$$

$$f(x) = 3x^2 + 3$$

The general quadratic that describes all members of this family could be

$$f(x) = mx^2 + m$$

1.   Is there anything that seems special about the values of *f*(*x*) for this family of functions?

If you can not think of anything, try evaluating some values of *f*(*x*) for a few functions.

Enter TABLE mode of the calculator. Use SET UP (SHIFT then MENU) to enter the set up screen for this menu. Ensure the settings are as shown below.

```
Variable    :Range        Plot/Line   :Blue     ↑
Graph Func  :On           Angle       :Deg
Dual Screen :Off          Coord       :On
Simul Graph :Off          Grid        :Off
Derivative  :Off          Axes        :On
Background  :None         Label       :Off
Plot/Line   :Blue    ↓    Display     :Norm1
 Rang LIST                 Fix  Sci Norm Eng
```

Press EXIT. Enter the six family members seen above.

```
Table Func  :Y=
Y1=-3X²-3
Y2=-2X²-2
Y3=-1X²-1
Y4=1X²+1
Y5=2X²+2
Y6=3X²+3
 SEL DEL TYPE COLR RANG TABL
```

Use RANG (F5) to set the values of x for which you want values of f(x) to be calculated to be as shown opposite.

```
Table Range
X

 Start:-10
 End  :10
 Pitch:0.5
```

Press EXIT and then use TABL (F6) to produce a table of f(x) values for the six functions.

```
   X     Y1      Y2      Y3
  -10   -303    -202    -101
 -9.5 -273.7 -182.5 -91.25
   -9   -246    -164    -82
 -8.5 -219.7 -146.5 -73.25
                        -10
 FORM DEL ROW      G-CON G-PLT
```

Examine the table.

2.   What is special about the values of f(x) for these members of this family of functions?

3.   Do you think that the whole family will have this feature?

4.   If you are still not sure, graph the values of f(x) that you have produced.

Press EXIT and then MENU and enter the GRAPH mode. Use SET UP (SHIFT then MENU) to enter the set up screen for this menu. Ensure the settings are as shown below.

```
Draw Type    :Connect
Graph Func   :On
Dual Screen  :Off
Simul Graph  :Off
Derivative   :Off
Background   :None
Plot/Line    :Blue    ↓
 Con Plot
```

```
Plot/Line    :Blue    ↑
Angle        :Deg
Coord        :On
Grid         :Off
Axes         :On
Label        :Off
Integration  :Gauss
 Gaus SimP
```

Press EXIT. You should see all of the functions you entered previously. There is a shorter way to enter a few members of a family of functions if all you want to do is draw a graph of the values of *f(x)* rather than produce a table.

Arrow down to Y7 and enter the following

$$mx^2 + m, [m=-3,-2,-1,1,2,3].$$

The m is entered by pressing ALPHA first, followed by 7. The **comma** key is just below the cos key. The **square** brackets are entered by pressing SHIFT then the **addition** and **subtraction** keys respectively. The = symbol is entered by pressing SHIFT then the **decimal** point key.

```
Graph Func   :Y=
Y3❚-1X²-1
Y4❚1X²+1
Y5❚2X²+2
Y6❚3X²+3
Y7❚MX²+M,[M=-3,-2,-1
Y8:
 SEL DEL TYPE COLR GMEM DRAW
```
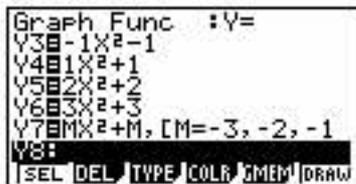
Select each of functions Y1 to Y6 in turn and use SEL (F1) to de select them so that they the equal sign does not have a dark square around it and hence will not be graphed (in fact they will but because of the general function you have entered as Y7)

```
Graph Func   :Y=
Y3=-1X²-1
Y4=1X²+1
Y5=2X²+2
Y6❚3X²+3
Y7❚MX²+M,[M=-3,-2,-1
Y8:
 SEL DEL TYPE COLR GMEM DRAW
```
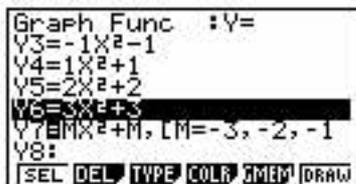
Press SHIFT and then use V. WIN (F3) to set the axes end points and scales as shown opposite.

```
View Window
Xmin  :-6.3
 max  :6.3
 scale:1
Ymin  :-10
 max  :10
 scale:1
 INIT TRIG STD STO RCL
```

Press EXIT and then use DRAW (F6) to draw the members of this family of functions.
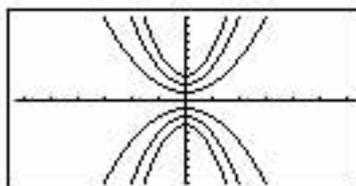
Does this make it clearer?

You can edit the function you entered and explore this situation for other values of *m*.

5.   What is special about the values of *f(x)* for these specific members of this family of functions?

6.   Do you think that the whole family will have this attribute?

7.   Make a conjecture about the family of functions $f(x) = mx^2 + m$.

8.   Prove your conjecture to be true for all *m* not equal to zero.

ANOTHER FAMILY

Consider the family of quadratics that have the following individuals (or specific quadratics) as members.

$$f(x) = -4x^2 - 2x - 1$$

$$f(x) = -2x^2 + 0x - \frac{1}{2}$$

$$f(x) = 2x^2 + 4x + \frac{1}{2}$$

$$f(x) = 4x^2 + 6x + 1$$

9.   Write down a general quadratic that describes the family from which these quadratic functions come.

10. Investigate the values of *f*(*x*) for the specific functions above and see if you can discover anything special about the number of zeros they have.

11. Document all of your findings and comment on whether or not you feel your findings may be generalised in some way to take into account all the members of this family.

12. Make a conjecture about the number of zeros this family of quadratics has.

13. Prove this conjecture true for all *m* values not equal to zero.

AND ANOTHER FAMILY

Consider the family of quadratics that can be described by:

$$f(x) = x^2 + (m+1)x + 1$$

14. Write down the specific members of this family for *m* = -5, -4, -3, -2, -1, 0, 1, 2 and 3.

15. Investigate the values of *f*(*x*) for the specific functions above and see if you can discover anything special about the number of zeros they have.

16. Document all of your findings and comment on whether or not you feel your findings may be generalised in some way to take into account all the members of this family.

17. Make a conjecture about the number of zeros this family of quadratics has.

18. Prove this conjecture true for all m values not equal to zero.

AND YET ANOTHER FAMILY

19. Make up a family of quadratics for which each member has two zeros.

A CHALLENGING INVESTIGATION?

Enter GRAPH mode and then use SET UP (SHIFT then MENU) to change Simul Graph from Off to On.

Press EXIT. If you have not done so already, define x^2 + (*m*+1)*x* +1, [*m* = -2,-1,0,1,2] as a family of functions in the calculator.

Press SHIFT and use V.WIN to set the end points and scales of the axes to the INIT (initial) (F1) settings seen opposite.

Press EXIT and then use DRAW (F6) to draw the graphs.

Sit back, wait and watch.

20.   Explore what is OH SO OBVIOUS a little further. Make a conjecture and prove it true for all *m*.

CALCULATOR SKILLS THAT MAY HELP

ZOOMING with a box

TRACING

G.SLV can be used to find intersection points (you cannot do this when functions are entered as a family. Enter them separately).

AND NOW FOR THE REAL CHALLENGE!

Consider the family of quadratic functions defined by $f(x) = x^2 + (m + 10)x + (m + 7)$.

There are many specific functions that belong to this family that are determined by the value of *m* that is chosen.

e.g.   if $m = 1$ the $f(x) = x^2 + 11x + 8$

Your job is to discover where these functions are situated on a Cartesian Plane.

Then build a spreadsheet (or whatever thing you like, *ANUGraph*, a graphics calculator, even your head and your hands) that will display the values of a reasonable number of these functions (say for $-10 \le m \le 10$ where $m \in Z$). Graph these functions (electronically is quicker).

Document any 'discoveries' you make, generalise them and prove them true for all functions of the form $f(x) = x^2 + (m + 10)x + (m + 7)$.

Now consider the family of functions $f(x) = x^2 + (3m - 1)x + (2m + 10)$.

Repeat the procedures that you have used to solve the problem above for this family. Use the same range of values for *m*.

Did you find a similar result for both families?

Attempt to prove your results from the above investigations are true for all functions of the form $f(x) = x^2 + (am + b)x + (cm + d)$. Put what you are attempting to prove into words.

Is the same result true for $f(x) = x^2 + (am^2 + b)x + (cm + d)$?

Is the same result true for $f(x) = x^2 + (am^2 + b)x + (cm^2 + d)$? Prove it.

The theory

Introduction

We are experiencing the most fluid time seen in the history of mathematics education. Many educational initiatives have come and gone and some have come back again, but few have caused the flurry of activity that has occurred due to the appearance of electronic technology (to be referred to as ET).

ET has been around for along time. It looked destined to never really take hold (I gave many professional development sessions to very few in the early 90s and most responses were, 'Great, but how do we get access to computers?'), until the real power brokers mandated its use in final examinations in some states. This has changed the game forever. The practitioners no longer had the choice of whether they would or would not. There was also money available to help with the cause. I guess this situation is not dissimilar to the changes Victoria saw when Common Assessment Tasks (CATs) were mandated. The moral of the story seems clear: if you want it to happen — mandate it.

Take a moment to think about the new teachers of mathematics who are young enough to remember being taught in a certain way, were trained to teach in a certain way (probably differently to the way they were taught) and have arrived at schools and are now being asked to teach in a manner that is likely to be different to what they remember and what they were trained to do.

Take a moment to think about some senior teachers of mathematics who are 'young' enough to remember being taught in a certain way, were trained to teach in that way and whose skills may not have evolved any where near enough for them to be equipped to cope with what is around the corner in South Australia.

Interesting times ahead you could say! We do need to work hard, debate the issues and try to resolve, as a nation, where we are and where we should be headed. What follows is one person's present thoughts about how the learning of mathematics can be enriched with the use of ET.

Why is ET so good? The dimensions of learning

Five years ago I could only explain why I thought the use of ET in the learning of mathematics was good by showing examples of the rich learning experiences that were possible to create. 'It helps students to develop to the full,' I would say. Not quite knowing what full meant. The pursuit of developing the mind is what learning is all about. The *Collins Australian Pocket Dictionary* defines learning as:

> *to get knowledge of or skill in by study, experience, etc.*
> *to come to know*
> *to memorise*
> *to acquire as a habit or attitude.*

It is, however, a lot more than this, but I always struggled to explain what. Then I was introduced to the Dimensions of Learning. It is not perfect, but DOL offers a very useful structure that helps when designing learning tasks, and a very helpful vocabulary for use when talking about learning.

The Dimensions of Learning is a synthesis of the learning process. Developed in America by Robert Marzano and his team, it offers a simple structure of what experiences are necessary if the mind is to be developed to its full potential. It is not subject specific. It also offers a vocabulary that is simple to relate to. There are five dimensions, one that centres around attitudes to learning and four being more academically focussed. They are:

- Positive attitudes and perceptions about learning

- Thinking involved in acquiring and integrating knowledge

- Thinking involved in extending and refining knowledge
  (comparing, classifying, making inductions, making deductions, etc.)

- Thinking involved in using knowledge meaningfully
  (decision making, investigation, experimental inquiry, problem solving, invention)

- Productive habits of mind
  (being sensitive to feedback, being accurate and seeking accuracy, working at the edge rather than the centre of your competence, being meta-cognitive).

Detailed information about the Dimensions of Learning and materials that are available can be found at http://www.mcrel.org/products/dimensions.

The dimensions are not meant to be, and should not be treated as, linear dimensions. The following diagram depicts the inter-woven nature of the dimensions.
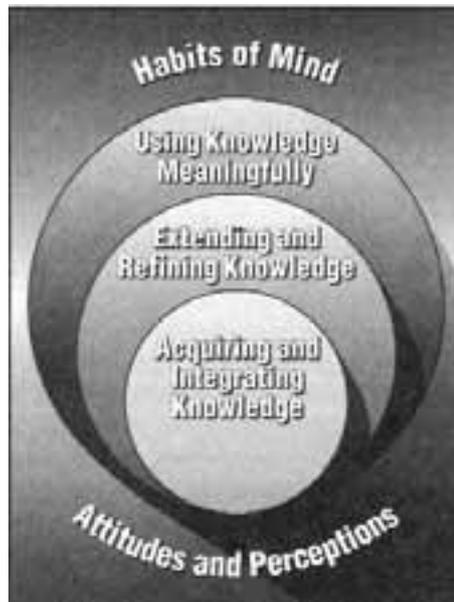
Figure 1: How the Dimension of Learning interact

The attitudes and perceptions with which our students approach their learning are as important as anything. Whether we like it or not kids have changed. They like gadgets with little screens — they learn from them. We now live in an interactive world in which kids want to interact. Most no longer see a compass and straight edge as intrinsically interactive and no matter what we do we will not change their view. An electronic compass, though — that is another story. However, if we believe that the ET version of the tool will

be lost on students if it is the first version of the tool they use, then we must ensure that that they first use the non-ET version of the tool. This is not an easy decision to make. It is hard to see things from a student's perspective and we, as teachers, often carry a lot of background and prejudice about these matters.

The type of knowledge students acquire can be categorised into two broad groups:

- factual knowledge that must be learned and be able to be recalled (declarative knowledge), e.g. 1/2 is a rational number;

- procedures, that need to be practiced, that when used will perform a task (procedural knowledge), e.g. the process of solving a linear equation.

With declarative knowledge, students must construct meaning and then organise and store the factual forms. With procedural knowledge students must construct, shape and internalise models of the processes they are asked to learn.

To learn in a rich manner one must accept that the knowledge we acquire should never be static in our mind. The initial learning of either declarative or procedural knowledge is rarely sufficient. We must offer students the opportunity to extend and *refine* their knowledge if they are to fully grasp its importance.

The notion of using the knowledge we acquire in *meaningful* ways is simply too logical to ignore.

To be a genuinely successful learner one must develop positive habits of mind such as testing the edges of their own ability, being aware of their own thinking, and being accurate and clear in all processes.

Connecting practice and theory

Let us first look at the good mathematics that comes out of activities like those seen at the start. It should be noted that these types of activities are harder to construct when one decides not to use ET.

The good mathematics:

- the idea of *one case of many* is heightened

- the ability to be *inductive* is heightened

- the ability to *make a conjecture about all cases* is heightened

- the understanding of the *concept of generality* is heightened

- the quest to *prove and understand what it means to prove* is heightened and can be *introduced at a much earlier point* in the student's career.

- the ability to generate something to hang your hat on when faced with very *abstract ideas* can be done simply and quickly.

It is possible to maximise the mathematical richness while minimising the pressing of buttons. As Rex Boggs would put it, we do not want to replace MSMs (mindless symbol

manipulators) with MBPs (mindless button pressers).

Blaming the tool for the loss of mathematical skill, understanding and the general ability to be mathematical is not sustainable. Like any tool, ET is open to abuse and no doubt the production of MBPs is a concern. However, this need not be the case if steps are taken to ensure the appropriate use of ET. But, what are the steps (or some of them)?

- Lots and lots and lots and lots of inservice, starting in the primary school and continuing up into the secondary school and beyond. If students are not exposed to appropriate methodology earlier enough, they will fail to gain from it later on. They will just want to be trained.

- A serious increase in the time teachers are given to prepare their lessons.

- Encouraging the writers of textbooks to think hard.

- A sharing, like there has never been before, of the materials of great teachers.

- Development of curriculum documents that allow the time for activities like those above to be realistically included in the learning experiences of students.

- Specific directives from those responsible for curriculum, re the balance between use of ET and non ET methods and what is considered appropriate and inappropriate use of ET.

- The development of assessment practices that encourage teachers to step their students through activities like those seen above.

Finally a brief comment about the costs and gains due the introduction of ET. The biggest cost as a result of a comprehensive introduction of ET will be time — both classroom and teacher. Some parts of traditional mathematical learning may be under threat of extinction in schools. Contrary to some opinions, loss of rigour need not occur, unless we let it happen. If it does happen then it is our fault, not the ETs.

The biggest gain, in a broad sense, will be students who are able to learn in the true meaning of the word, as outlined in the Dimensions of Learning. They will know how to and be able to work mathematically. In short a reduction in the number of MSMs and an increase in the number of PSMs (purposeful symbolic manipulators).

ET will not change the fact that mathematics is largely concerned with the manipulation of symbols to reach a desired end point. It will, however, help students to take the steps that illustrate reason for the manipulation, in a way they were rarely able to do before. It may also eventually change how we perform the algebraic acts, but the acts will still be performed.

So let us not blame the tool for the ills we see around us. Rather, let us work hard to ensure the enormous potential ET promises is realised as — let us be realistic — ET is not going to go away!

Summary and Reflections

Kim Wright

Northern Territory Department of Education

Introduction

Tony works at Prince Alfred College as the Mathematics Senior. The session contained both a practical lesson and an informative talk about the history of graphic calculators used at Prince Alfred College. Tony discussed the theory and practice of two lessons; a Year 9 and a Year 11 lesson as portrayals of good strategies for utilising graphic calculators into the classroom.

Year 9 lesson

The difference between any pair of consecutive square numbers.

Background knowledge of students: Year 8 students study patterning algebra and concepts such as the distributive law.

The steps of the lesson are:

Seeing   the students can see the pattern in the numbers once they have found a few pairs of consecutive numbers and the differences between them. The class amasses the data from the class and use the table function on the graphics calculator the find the difference between many pairs on consecutive pairs.

Conjecture   the students make a conjecture on the results in words, e.g. they are odd numbers.

Conjecture   the students make an algebraic conjecture using algebra patterning, e.g.

Discussion   Does the pattern break? Using the graphics calculator or a spreadsheet the students evaluate specific cases.

Proof   Reason to move on!

Extension   Extension questions are provided for the more able students.

Each student in the Year 9 class could cope with the first ten questions and only the best students could complete the extension.

Stendusers (activities that STarts a topic, ENDs a topic and hopefully enthUSEs the students along the way) often require the use of some form of ET (electronic technology). They are meant to be simple enough for the students to start on, hard enough so that they can not solve them without learning some new theory and interesting. They may be worked on throughout the topic of study and can be recalled at any time to help pursue new knowl-

edge. The final solution of a Stenduser normally requires students to learn in a manner discussed in the higher dimensions of learning.' (Tony Harradine) [PAC stands for Prince Alfred College]

Tony then used a harder example to show that 'the product of four consecutive numbers plus one always gives a perfect square'.

Year 11 lesson

What are all those quadratics doing?

Tony compared his technology-based approach — with an emphasis on looking for families and functions — with what the textbook questions that most students currently do.

Key issues raised

Tony discussed the following issues:

- *Is this good mathematics?*

  The school works on lots of activities using technology.

  The benefits:

  - this is perhaps the most fluid time mathematics education has seen;

  - mandating the use of Electronic Technology (ET) has forced the hand of many;

  - teachers have expanded range of ways of viewing things.

- *The Dimensions of Learning*

  This framework helps emphasise attitudes and thinking important in mathematics, and electronic technologies fit well with it.

Conclusion

The erosion of skills is not the electronic technology's fault! Such tools are open to abuse.

To improve we need to have

- inservicing

- increase in teachers' preparation time

- textbook writers encouraged to think

- sharing amongst teachers

- curriculum changes to allow time to complete learning

- assessment practices that value complete learning.

People can see value in teaching with the use of electronic technology but the curriculum needs to reflect it. The emphasis is still too much on having to finish teaching the content. Teachers also need an in depth knowledge of mathematics. They also need to get over the thought that graphic calculators are just a fancy scientific calculator or just an alternative to computers.

Seeing is Believing

Michael Hutley

Geelong Grammar School, Victoria

It is my conjecture that students of today are more familiar with the communication of information of a dynamic visual nature, than students of yesteryear. It is not sufficient to simply provide them with a picture; they want to be able to interact with it. Not only are they comfortable with this medium but they expect to use it, and demand it now. They have developed the skills necessary to interpret and investigate what they see. A graphic calculator is another mode that can quickly furnish this type of information. Failure to provide mathematics in a dynamic visual form denies today's students their opportunity to truly understand the core concepts.

To support this, this presentation discussed a variety of classroom anecdotes that cover a wide range of mathematical concepts. Most of these will be from the senior years as this is where the graphic calculator is currently being used, in itself something worthy of discussion. I must point out that while I feel that my experiences are more relevant than an anecdote, they are none the less just that: my experiences. I have not conducted a detailed study or analysis of grades that might provide more concrete evidence. The concepts covered range from simplification of surds, curve sketching, transcendental functions to inverse functions. I hope therefore that my experiences answer some questions and provoke others.

Adding value to student's conceptual learning

What does this mean? In a mathematical context 'value' can be thought of as magnitude. This may suggest only a quantitative gain, however I also feel that there is a natural qualitative gain associated with this. Through using graphics calculators, my students see more examples, have the opportunity to change variables more often and more readily. They pose conjectures more often and test them more frequently. My experience over the last five years supports the notion of 'Adding Value'.

Graphics calculators at Geelong Grammar School (GGS)

We in the GGS Mathematics department feel that technology has increased the number of opportunities that students have in which they conceptualise. In the past students have required strong arithmetic and algebraic skills to have the opportunity to access mathematics in the senior years. A lack of these skills virtually prohibited a student from the process of forming an idea from mathematical experiences. Students were voting with

their feet, by opting to do other subjects. With the introduction of graphing calculators mathematics is more accessible to the 'weaker' student. This is not to suggest that graphing calculators are only of value to weaker students. I hope to show examples where our more capable students have been able to add to the number of concepts they consider when compared to previous students in the same year level.

A brief history of the implementation of graphing calculators at GGS may be useful to those just beginning to use them in the class room.

1995   3 class sets of SHARP EL9300.
Rationale: 'to familiarise staff through use in class';
                   'to promote thinking about changes in pedagogy'.
Outcome: limited use.

1996/7   All Year 11 and 12 students buy their own graphing calculator.
Rationale: 'allowed in external exams but not an advantage'
Outcome: teachers started to use them to answer exam questions.

1998   1 OHP unit.
Outcome: leading edge teachers begin to use graphing calculators as a learning tool.

1999   New graphing calculator, SHARP EL9600 and 4 OHP units.
Teachers of the Further Mathematics course decide to take on board the graphing calculators as a learning tool.
Leading edge teachers use them all the time.

2000   6 more OHP units: one for each Year 11 and 12 teacher.
Even the most experienced teacher uses it: 'I use it all the time', 'Best result ever in recent test.'

As mentioned above the Further Mathematics teachers were the first to embrace the graphing calculators. In 1999 the school placed second in the state in Further Mathematics. 56% of our students finished in the top 8%, compared with 29% the year before.

Five classroom examples

Seeing is believing

A simple illustration to me of the power of seeing occurred in my Year 10 class. The concept was multiplication of surds. I started with $\sqrt{2} \times \sqrt{3}$ by using the OHP unit. The students saw the question and the answer in decimal form. I then asked students who had never multiplied surds to suggest what they thought the exact answer would be? $\sqrt{6}$ was then shown on the OHP. The students could see both questions and answers.

I then asked $\sqrt{2} + \sqrt{8}$ to which to response was $\sqrt{10}$. The students could see that this was obviously wrong. When I typed $3\sqrt{2}$ they were truly amazed. It prompted a most lively discussion.

Programming options

In Further Mathematics we look at the Sine Rule. The graphics calculator is capable of being programmed to find an unknown given three known values. You may ask, 'Is this learning?'. My response is that the concept being tested is which three pieces of information are required.

Example:
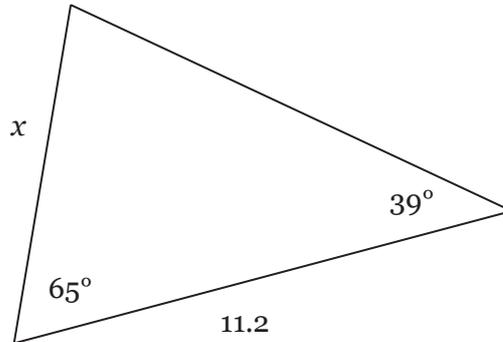
As can be seen, the student needs to decide what information is required and must correctly identify which angle is opposite which side, by correctly labelling the diagram. Once this has been done the student is assured of the correct answer through the use of their graphing calculator. They are not frustrated by their lack of algebraic skills.

Is that point on the line?

A concept that one may assume is covered in Year 9 is whether or not a particular point is on a line.

I posed the question 'Is the point (1, 3) on $3x + 4y = 12$?' to my Year 11 class. I was deeply disturbed by their response. I showed them that by substitution that $3(1) + 4(3) = 15$. Unfortunately this did not enlighten many. I then went to the graphing calculator OHP unit. Problem: it only accepts $y = mx + c$. After a little algebraic manipulation I plotted and graphed $y = \dfrac{12 - 3x}{4}$. I then used the trace function and asked students to substitute each pair of coordinates into the expression $3x + 4y$. They quickly got the picture. Particularly when we traced to the point (1, 2.25).

Broadcast learning

One of the options that a graphing calculator OHP unit allows is to enhance the quality and flexibility of broadcast learning. Again in Year 11 Mathematics Methods, we were able to investigate the relationship between repeated factors in polynomial functions and the nature of the x-intercepts.

Students watched the following functions being graphed.

$Y1 = (4 - x)$
$Y2 = x(4 - x)$
$Y3 = x^2(4 - x)$
$Y4 = x^3(4 - x)$
$Y5 = x^4(4 - x)$

At the students' request I used various functions of the graphing calculator such as TRACE, ZOOM, and JUMP. This enabled the students to investigate the properties of each graph.

They were then asked to compare and contrast each of the five graphs. After some discussion a conjecture was formed. I then asked what would the following graphs look like?

$$Y6 = x(4 - x)^2$$
$$Y7 = x(4 - x)^3$$
$$Y8 = x(4 - x)^4$$

The students sketched what they thought the graphs would look like and checked their answers with their graphing calculator.

Then next lesson they investigated graphs of $y = (A + x)^m(B - x)^n$. This type of investigation is not something that students in their first term of Year 11 have done before. However, they produced excellent ideas and gained a high level of appreciation of the effect that a repeated factor has.

Student generated learning

Understanding the concept that $a \sin x + b \cos x = r \sin (x + \alpha)$ is greatly enhanced through the use of a graphing calculator. A colleague of mine, Clive Moffat, produced a brilliant student generated learning assignment for students studying Advanced General Mathematics. Not only did the students look at this concept but extended it to $a \sin nx + b \cos nx = r \sin n(x + \alpha)$ and then to $f(x) = a \sin nx + b \cos mx$ where they then modelled the effects of tidal waves and wind and waves on a boat moored to a jetty.

Students started by graphing $y = 3 \sin x + 4 \cos x$. They quickly realised that the resulting graph was in fact another sinusoidal curve with a period of 2π, but greater amplitude and an x-translation.

Conclusion

While five classroom examples are never going to be sufficient to conclusively prove any conjecture, I hope that they do lend it support. Students tend to believe what they see. Students of today are used to receiving information visually and become easily engaged with visual information. A graphing calculator can remove some of the barriers to mathematical investigation and appreciation. Graphing calculators allow students to quickly manipulate a set of conditions and receive prompt visual feedback. This enables students to investigate mathematical situations that in the past were beyond their reach. Graphing calculators certainly seem to add value to student's conceptual learning.

Summary and reflections

Howard Reeves

Department of Education, Tasmania

Presenter notes

Michael Hutley is Head of Mathematics at Geelong Grammar School. He has been a teacher of Mathematics in the secondary Years 7 to 12 in government and non-government schools in Victoria for seventeen years. Two years ago Geelong Grammar introduced a laptop computer program for all students.

Michael Hutley's presentation commenced with some scene setting with respect to the introduction and use of graphics calculators at Geelong Grammar School. He described the graphics calculator 'journey' at GGS beginning in 1995 with the purchase of three class sets, while still holding on to many pen and paper techniques.

The initial use of the calculators was in Year 11, 'principally because the school was a notebook school'. When the calculators were used in 1996–7 in the school examinations their use was in ways 'to do the examinations'. By 1998 teachers were beginning to use the calculators 'all the time'. The older and experienced Mathematics teachers in the school were reluctant to use the calculators but made the shift via the use of overhead projector display units for demonstration purposes.

When it came to success with the use of graphics calculators, Michael discussed their use in Further Mathematics, where on a state-wide basis Geelong Grammar School students performed very well (56% of the GGS students finishing in the top 8% in 1999). He believes that this is attributable to greater access to the mathematical ideas and concepts of the course by the students and the increased visualisation skills of the students brought about in both cases by the use of the calculators.

For the remainder of the Master Class, Michael set about providing evidence in support of his conjecture '… students of today are more familiar with the communication of information of a dynamic visual nature, than students of yesteryear'. In support of the conjecture he provided examples under five headings:

Seeing is believing

In the study of surds, for example, seeing that $\sqrt{2} \times \sqrt{3} = 2.4494897…$ and $\sqrt{6} = 2.4494897…$

Similarly seeing that $\sqrt{2} + \sqrt{8} \neq \sqrt{10}$ but $\sqrt{2} + \sqrt{8} = 3\sqrt{2}$ by looking are the decimal approximations.

In function study, seeing the translations of the graph of $y = x^2$ with different values of a, b and c in $y = a(x+b)^2 + c$.

Programming options

The example provided here was a solution of triangles using sine/cosine rules. The mathematics is in labelling the diagram, selecting the appropriate formula etc. rather than the final calculation step. Students write and enter a program for the calculation of sides and angles. Students entering their own program is preferred to downloading a program.

Is this point on the line?

Using the graphics calculator to investigate in a variety of ways questions like:

> *Is (1,3) on the line 3x + 4y = 12 ? Draw the graph and use the [TRACE] facility or construct a table of values.*

Broadcast learning

Using the graphics calculator overhead projection units for the teacher to lead investigations with students. The example Michael presented was from function study, investigating multiple zeros in $y = x^n(4 - x)$ and more generally $y = (a + x)^m(b - x)^n$. 'Let the students decide which functions they will graph next.' 'Students get a better and earlier understanding of graphing functions with a graphics calculator than without one.' A remark from the floor was 'Inexperienced Mathematics teachers do too!'

Students generate their own learning

The graphics calculator puts in the hands of the learner the power to generate their own learning with examples beyond their algebraic capabilities, for example in extending the study of $a\sin x + b\cos x = r\sin (x + a)$ to examples like $3\sin 3x + 4\cos 3x$.

In summary, this interesting and informative Master Class highlighted two issues of significance for teachers and students:

- graphics calculators give students greater access to some of the more challenging mathematical ideas;

- graphics calculators increase student participation in the some of the more demanding Mathematics courses.

Graphics Calculators and Assessment in Mathematics Education

John McKinlay

Brisbane Grammar School, Queensland

An introductory example

Examine the graph shown in Figure 1.
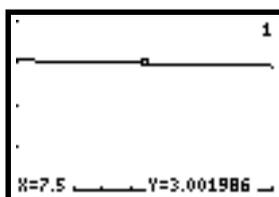
Figure 1

Hardly the most exciting graph you have every seen is it? Yet its interpretation proved to be a rich assessment task for students in a Year 11 class who had explored a number of topics within financial mathematics, including the compound interest formula. Here is the question they were asked:

Enter the function $y = \left(1 + \dfrac{x}{100}\right)^{\frac{114}{x}}$ into your graphics calculator.

(a)   What is an appropriate range given the domain of: $3 \le x \le 12$?

(b)   In light of your studies of the 'rule of 72', explain/interpret the graph of this function over this domain.

The 'rule of 72' basically states that if you divide the current interest rate into 72, the number you get gives the approximate number of years it will take for an investment to either double in value (in the case of appreciating investments) or to halve (in the case of depreciating investments). Since for any principal, P, the amount to which it grows in $72/x$ years will be 2P, (for an appreciating investment), the compound interest formula simplifies to:

$$2 = \left(1 + \frac{x}{100}\right)^{\frac{72}{x}}$$

This rule of 72 is confined to interest rates of about 3% to 12%. Hence the graph of this function over this domain gives the flat line graph of $y = 2$ as shown in Figure 2. Not really so dull though! Hence, students who met the graph shown in Figure 1 in the context of a financial mathematics test item, with the prompt of the rule of 72, could be expected to

recognise that there might be a 'rule of 114' whereby 114 divided by the interest rate would give the approximate number of years for an investment to *triple* if the investment were appreciating in value.
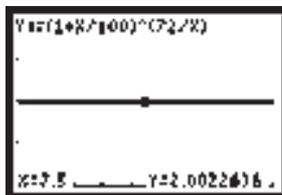
Figure 2

This example illustrates my first point: if students have been expected to use graphics calculators in their learning experiences then it is crucial that they be expected to use them in assessment.

Assessment can be motivating for students!

No surprises here you think: inform students that they are going to be assessed on what you as the teacher are covering in a lesson and you just might motivate the rascals to participate a little. This is not the motivation I am referring to. I am referring to an assignment task begun in class that so motivated a class of Year 10 students that staff were forced to work into morning tea because the students did not want to stop working! After the assignment was completed, the students thanked their teachers for setting the assessment item and requested others like it!

The investigation, adapted from *Graphic Algebra* (Asp, Dowsey, Stacey & Tynan, 1995), required Year 10 students to use a graphics calculator to create patterns similar to those shown in Figures 3 to 5 using the graphs of quadratic functions. The skills of determining the general form of the equation of a quadratic and the effects of the values of the constants a, b, and c on the graph were vital to this task.

Note that a similar investigation was initially run with linear functions.
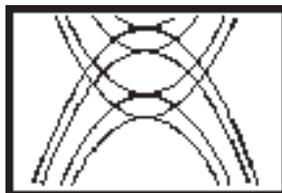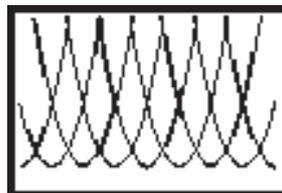
Figure 3   Figure 4   Figure 5

In his text *Assessing Students: How Shall We Know Them?* Rowntree (1977) identifies three major purposes of assessment: to motivate students, to provide feedback to students, and to provide feedback to the teacher.

This assessment task met these purposes. The students were highly motivated, they gained immediate feedback on the success or otherwise of their proposed functions, and the teachers gained valuable insights into the misconceptions of the students regarding the graphs of quadratic functions as they toured the classroom and attended to students grap-

pling with the task. There was an interesting flow-on effect with this task as well: teachers who were slow to administer the task were actually admonished by their students who had heard from their peers in other classes that this 'terrific' maths investigation was to be done! This leads me to my next point.

Assessment can be motivating for staff!

After attending a workshop on the use of graphics calculators, two staff returned full of missionary zeal. Their enthusiasm and shear doggedness regarding how students in the Year 9 cohort just had to be assessed convinced the most recalcitrant of their teaching colleagues. The assignment they coerced all to do is summarised below. The topic was coordinate geometry and the activity was adapted from *The Sub-AToMIC Project*.
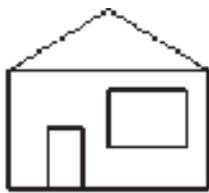
Instructions for drawing the picture of the house shown at left using a graphics calculator were given to a Year 9 class, who then had to replicate the drawing of the house and then create their own picture, first on graph paper and then using the graphics calculator.

The house can be created using the graphics calculator by entering the coordinates of the points that must be connected and then creating a line segment graph. The house was created by joining the following points in the order given below.

HOUSE (inc roof + door)		LINE ACROSS ROOF		WINDOW	
x coord (L1)	y coord (L2)	x coord (L3)	y coord (L4)	x coord (L5)	y coord (L6)
-2	-4	-5	2	2	1
-5	-4	3	2	2	-2
-5	2			-1	-2
-1	5			-1	1
3	2			2	1
3	-4				
-2	-4				
-2	-1				
-3.5	-1				
-3.5	-4				

Examples of some of the pictures created by the students are shown in Figure 6.

Figure 6

This investigation illustrates another feature of the use of graphics calculators in the assessment of mathematics: they provide a means of injecting new life (modernising if you like) old ideas that have proved valuable in the past.

The use of assessment to link topics

Graphics calculators have assisted enormously in breaking down the stereotyped view of mathematics as disjointed strands or topics. The example offered below illustrates a connection that students can be coaxed to make between financial mathematics and a seemingly unrelated mathematical number *e*. The task was included in a class test administered to Year 12 students who had had the use of graphics calculators through a hire scheme since the start of Year 11.

Task

- Calculate the final amounts A1 to A6 if $1 is invested for 1 year at 100% pa compounding with the following rests:

  (i) annually   (ii) semi-annually   (iii) quarterly

  (iv) monthly   (v) weekly   (vi) daily

- Determine a general formula for this situation, to calculate A given the number of rests, *n*.

- Describe and interpret the graph of this function for $0 \le n \le 365$.

Solution

A1 = $2; A2 = $2.25; A3 = $2.44; A4 = $2.61; A5 = $2.69; A6 = $2.71

The general formula is: $A = \left(1 + \dfrac{1}{n}\right)^{n}$

The graph of this function over the recommended domain (Figure 7) is provided in Figure 8.
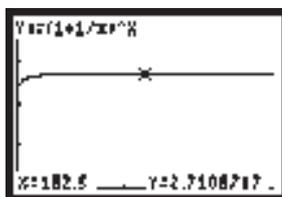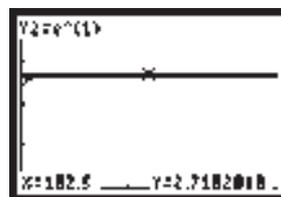
| Figure 7 | Figure 8 | Figure 9 |

From the graph, there is a limit as *n* approaches infinity of A = 2.71. This means that the original $1 invested will never grow to more than this no matter how many compounding rests. Furthermore, this limit approximates the number *e*. The graph of e^{1} has been drawn over the graph of the general formula of *A* in Figure 9. As can be seen the graphs match.

Therefore, one can argue that $\lim\limits_{n \to \infty}\left(1 + \dfrac{1}{n}\right)^{n} = e$.

The use of graphics calculators to assess real-life applications

Graphics calculators have provided a tremendous opportunity for mathematics teachers to explore real-life data instead of data that has been massaged or adjusted to ensure that algorithms can be applied easily or to allow an 'ideal' solution to a problem to be attained. One of the advantages of using real-life data in assessment items is that quite often a task will have more than one solution, as is demonstrated below.

This assessment item was part of an assignment given to Year 12 students on the topic of exponential and logarithmic functions. The task required students to examine the cooling power of the wind as a function of wind speed and to determine a model that best fitted the data provided. They were also required to examine the limitations of their model. Students were encouraged to work together on the task but the submitted work had to be their own. Justification of the algebraic model chosen to fit the data was to be supported by downloaded calculator screen 'snapshots.'

The data provided to the students was obtained from the Bureau of Meteorology and the task was used in The AToMIC Project.

Wind Chill Effects at 8° C

Wind Speed (km/h)	6	10	20	30	40
Equiv. Air Temp (degrees C)	7	5	0	-3	-5

The equivalent air temperature, *T*, was to be expressed in terms of the wind speed, *S*. The resultant equation was then used to check the equivalent air temperatures for wind speeds of 6, 10, 20, 30, and 40 km/h. Students were also required to determine where their model broke down, that is, they had to cite examples of where the equivalent air temperature was obviously incorrect for a given wind speed.

Possible solutions

There are a number of different approaches to this problem and a number of suitable models that can be applied. Two will be demonstrated here. Note that the first is from a teacher and the second is from a student. The second is possibly more elegant!

Approach 1

Using the data as presented is not appropriate, for although there appears to be an exponential decay function here, the negative values for some temperatures prevent such a model being applied. So, if an arbitrary value of (say) 10 is added to each of the temperatures, the following model can be obtained (Figures 10 & 11).
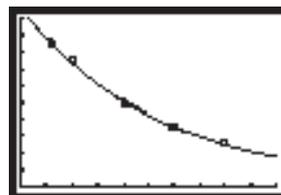
Figure 10   Figure 11

Hence this model for wind chill would be $T = 21.184 \times 0.964^S - 10$. Using this equation, the predicted temperatures for the various wind speeds match very well (Figure 12).

The model breaks down when the wind speed drops below 4 km/h. As Figure 13 shows, the model predicts that the temperature will *increase* instead of remaining at 8° C.
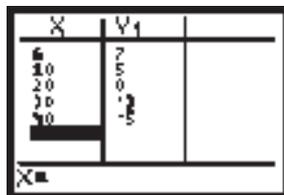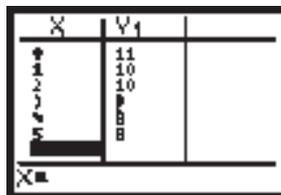
Figure 12   Figure 13

Approach 2

Another approach is to consider the wind speed versus the temperature drop (Figure 14). One model which fits this data well is a quartic (Figures 15 & 16).
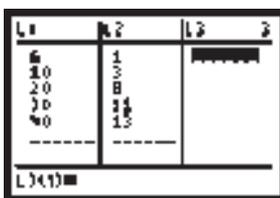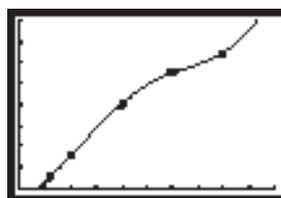
Figure 14   Figure 15   Figure 16

Again, the predicted values match very closely (Figure 17) and again the model breaks down at wind speeds below 5 km/h. When the wind speed is 0, for example, instead of the temperature being 8° C, the model predicts the temperature will rise by 1° (Figure 18)!
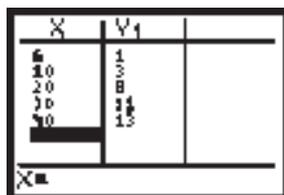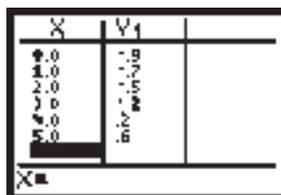
Figure 17   Figure 18

In her keynote address at the 1999 AAMT Biennial Conference, the President of the NCTM, Glenda Lappan spoke of the recently drafted NCTM *Standards* documents (Baldwin & Roberts, 1999). The *Standards* vision is based on a set of commitments to the teaching and learning of mathematics that include the following:

Inclusiveness. Effective mathematics teaching and learning should be experienced by every student with the commitment to developing mathematical power for all students.

Depth over coverage. Curriculum, instruction and assessment should emphasise a smaller number of big, powerful ideas.

Understanding. Curriculum, instruction, and assessment should be aligned to foster genuine understanding of the big ideas and processes in mathematics.

Active engagement. Genuine learning requires active mental and, at times, physical engagement, and efforts to understand and use ideas.

Investigations. Deep understanding is best promoted by posing problems and questions, and then skillfully guiding problem-solving and discourse so that students' ideas are constantly probed and pushed toward more powerful mathematical realisations.

Application. Curriculum, instruction and assessment should foster an ability and a disposition to use mathematical ideas and processes flexibly to solve non-routine problems which come from real world contexts as well as from mathematics itself.

Connections. Deep understanding involves making connections between one's informal knowledge of mathematics and more formal mathematics and mathematical language of the discipline as well as among areas of mathematics and between mathematics and other disciplines.

Note that curriculum, instruction *and* assessment were considered inseparable in many of the above facets. One can see within the assessment task on the Wind Chill data an endeavour to fathom student *understanding* of exponential functions in a *real world context* via an *investigation*. This is consistent with the vision embodied in many Australian curriculum documents and the NCTM *Standards* (NCTM, 1999). There is an obvious *connection* with the discipline of meteorology. Note too that the task has an open-ended nature and that no student could sensibly progress towards a solution to the problem without the use of a graphics calculator or some other technology.

The task was designed to reflect student learning experiences, was motivating for students and staff, and linked the mathematical topics of statistics and exponential functions.

Shared assessment tasks

Mathematics supposedly provides students with skills that they can apply in many other subjects. A strong case can be made for mathematics teachers to look outside our discipline's walls and explore the application of mathematics in subjects such as science, accounting, home economics and health and physical education. Furthermore, many of our students are heavily burdened by assessment. If one assessment task could give a student credit in more than one discipline this burden might be eased somewhat. An example of an assessment task that was designed for a senior mathematics and physics class is offered below.

The mathematical topic being assessed was periodic functions. The task was administered to Year 11 students as a single class test item — they had fifty minutes to offer a solution to the problem. The students' graphics calculators had been collected the night before the test and the same data downloaded to each. The lesson included a brief demonstration of how the data were collected. Although the assessment task guided the students in how to obtain the graph shown in Figure 19, any student who had difficulty achieving this was given assistance. Students were being assessed in their analysis of the graph, not in their ability to procure it.

When a tuning fork vibrates, it disturbs nearby air molecules, creating regions of higher-than-normal pressure and regions of lower-than-normal pressure. Tuning forks are used by piano tuners when tensioning piano strings to the correct pitch (or note). A set of tuning forks consists of one octave (C to C'):

Pitch	C	D	E	F	G	A	B	C'
Frequency (Cycles/second)	256	288	320	341.3	384	426.5	480	512

These pressure variations were recorded using a microphone, a graphics calculator, and a Calculator Based Laboratory (CBL) system. The resulting curve is a periodic function that can be analysed. Students were required to determine a trigonometric function that modelled the data.
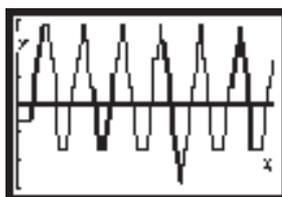
Figure 19

Told that the frequency is the inverse of the wavelength, the students were also required to determine the pitch of the tuning fork used to generate the data. After all papers had been collected, the students could ask to see the tuning fork used. Since each tuning fork has the frequency stamped on it, the students gained immediate feedback regarding their solution!

Assuming that the function is of the form $y = a \sin(nx + c)$, one can determine values for the constants. Any errors are immediately apparent because the graphed function will not match the plotted data shown in Figure 21. The amplitude, a, was approximately 0.014 (Figure 20). The wavelength was approximately 0.004 (Figures 21 and 22). The frequency was therefore approximately 250 — matching a tuning fork of pitch C.
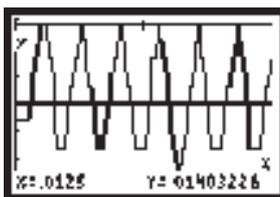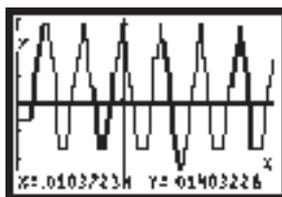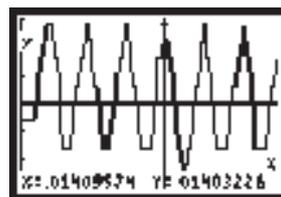
Figure 20	Figure 21	Figure 22

Three observations worth noting with this assessment item are: (i) some students, accustomed to neat textbook values for the constants a, n and c, were really thrown by the 'messy' values of this data; (ii) some students proved incapable of determining any of the constants at all; and (iii) some students, upon being shown the tuning fork that was used to collect the data, moaned that they had 'stuffed it' since they had got a frequency of 250 but the fork read 256!

So far, this paper would appear to suggest that many of the problems associated with assessment in mathematics can be remedied by incorporating graphics calculators into the curriculum. Students and staff are motivated, old ideas can be given new life, and real data can be modelled efficiently, and we can even justify it through reference to the NCTM

Standards or any number of curriculum documents published in this country. Where is that fly in our ointment?

Some problems are magnified...

So, graphics calculators are in the hands of your students and the staff feel confident enough to use them for the basic stuff — function plotting, entering and analysing data, applying regression formulae, matrix operations and the like. As the mathematics staff mentor on graphics calculators, you anticipate their implementation will be like Figure 23 — steady acceptance and application, almost exponential as they are converted!
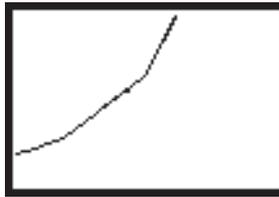
Figure 23

However, you are prepared to admit that a couple of staff members are taking a little longer to adapt to the new technology that you want to utilise, so you are prepared to allow some to progress at a slower rate (and even regress at times) — something like Figure 24 perhaps.

Figure 24

Then, when you suggest that since students have all been using the graphics calculators in class then they should use them in assessment and that the assessment items should assume that students are proficient in the use of the devices, some mathematics teachers confess that they have not been as enthusiastic about them as you have been and that their students would be disadvantaged — your grand plan is looking more like Figure 25!
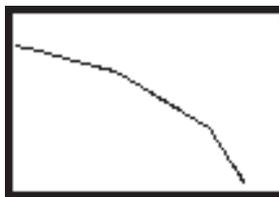
Figure 25

Each of the graphs shown in Figures 23 to 25 were obtained by 'zooming in' on the graph shown in Figure 26. If you were able to 'zoom out' and examine the views of all the staff, you would see that the introduction of graphics calculators has mirrored the diverse philosophical beliefs of the mathematics staff regarding curriculum and pedagogy.

Figure 26

When the syllabus guides teachers to 'model linear functions' and everyone has the same text exercises to complete, such differences are easier to conceal than if a graphics calculator can be used. There are many ways of modelling linear functions using a graphics calculator (including not using the calculator at all!). Some ways might be 'better' than others for a particular class but there is no one way. However, some staff members may not want to consider that there are alternative ways of teaching a mathematical concept or skill. Some are innovative and some have innovation thrust upon them? As has been the case in the past, sometimes the assessment tail has to wag the curriculum dog.

It is relatively easy to recognise the beneficial role graphics calculators can have in the assessment of the curriculum topics illustrated in this paper so far. But one person's comfort zone is another person's pain zone...

Some problems are new...

I will conclude with a quiz. I suspect that many may identify with the trend shown in Figure 23 above but where would you draw the line? For each of the statements in the left hand column, readers are invited to indicate their preference along the continuum at right. The quiz only targets Years 7–10 (an often neglected sector for implementation of this graphics calculator technology).

Junior students should be able to use a graphics calculator to:

	Never — Always
Sketch linear functions	
Sketch quadratic functions	
Find the roots of quadratic functions	
Perform linear regression	
Perform quadratic regression	
Calculate the mean of a set of raw data	
Calculate the mean from a frequency table	
Calculate the median of a set of raw data	
Calculate the median from a frequency table	
Construct a histogram	

	Never	Always
Write and run simple programs	←	→
Download programs from the net	←	→
Expand and simplify algebraic expressions	←	→
Factorise algebraic expressions	←	→
Solve equations	←	→

The list above is not meant to be exhaustive. The exercise to intended to generate discussion. There are graphics calculators available that will perform all of the above functions and as mathematics teachers we have some new problems to face now: what 'paper and pen' skills do we value and what are we willing to de-emphasise? Some schools still teach the square root algorithm and how to calculate the correlation coefficient. Others show students how to do these calculations quickly using scientific calculators. Instead of assessing the *calculation* of the correlation coefficient, such schools assess student ability to *interpret* and *apply* it.

What do you currently assess that a graphics calculator can do more efficiently? Should you be assessing your students' ability to interpret, analyse or apply these skills?

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Summary and Reflections

Cheryl Meade

Curriculum Council, Western Australia

I had the privilege of attending the presentation given by John McKinlay from Queensland, simply entitled 'Assessment'. While the title might have been simple I found his presentation both informative and thought provoking. John's easy style gave you an insight into how exciting the mathematics journey would be as a student in his class.

Using a number of assessment examples, both formal class tests items and investigative items, John argued that:

- if students have been expected to use graphics calculators in their learning experiences then it is crucial that they be expected to use them in assessment;

- assessment items, if carefully chosen, can actually be motivating for students — not in the sense that they are 'driven' to work but that the right task can engender an enthusiasm for mathematics;

- some assessment items can also be motivating for staff — it is not always necessary to invent new tasks simply because we have new technology: 'old' ideas can be given a new lease of life by simple tweaking using graphics calculators;

- assessment can be used to link topics, given the ability of the graphics calculators to handle data as well as functions: he gave us an example of a task which coaxed students to recognise a link between a compound interest case, the number e and limits;

- assessment can use real-life data where students can see the application of the mathematics that they are studying. The task does not have to provide clinically altered information to ensure that the answers are 'neat'. Students are able to recognise that sometimes there may be more than one solution to a problem. The graphics calculator can assist them to justify their solution. The anecdote he shared through this example gave valuable insight into the value of risk-taking with our students so that both parties benefit!

John referred to the NCTM standards and pointed out that many of his cited assessment items fulfilled the requirements of depth over coverage, understanding the 'big ideas', active engagement, investigation, application and connections between mathematical ideas and connections with other disciplines.

He further pointed out that some assessment tasks could be used by the mathematics department and other subject areas such as physics, accounting, health & physical education. A valuable strategy for helping students overburdened with assessments!

Some problems to do with assessment are magnified with the day-to-day use of graphics calculators. The differences in pedagogy may not be obvious when teachers commonly rely solely on texts. However, the range of mastery and application of graphics calculators by

staff can lead to great differences in learning experiences for students. This obviously impacts on assessment given John's first point.

John then shared with us a quiz which clearly illustrated that we are all in different comfort zones in regard to the application of graphics calculators to common mathematics topics. We now have to decide to what extent are graphics calculators going to supplement or replace current topics that we have long 'treasured'.

Discussion following his presentation centred around three main points:

- a concern that graphics calculators could lead to too much of an empirical emphasis when there was a need to engender in students the ability to prove and justify as well;

- difficulties associated with assessing students on the criteria required for two separate assessment systems e.g. when the International Baccalaureat and a state system are integrated at the same school; and

- the importance of sharing of resources associated with the implementation of graphics calculators.

Certainly in WA, it has been the culture of sharing amongst mathematics educators that has enabled a relatively smooth introduction of graphics calculators into our external examination. As people developed expertise this was openly available to other teachers across system/sector and post-school destination boundaries. Ultimately, I believe that we cannot over-emphasise the need for supporting each across school and state boundaries.

The Graphics Calculator: a V.A.T. (value adding technology) in the Mathematics Curriculum

How graphics calculators add value to our knowledge of student learning through changed assessment practices

Gary O'Brien

Cannon Hill Anglican College, Queensland

Background

The graphics calculator may be the major advance in mathematics education in the last twenty-five years. It has the potential to revolutionise what we value in a mathematics curriculum. The introduction of the graphics calculator can, in many ways, be likened to the goods and services tax. To many it provides the opportunity to rethink the things we value and the way we do things; it provides the chance to change our practices and rethink the way we operate. It may provide the chance to forge a different path ahead. On the other hand, there are the rumours, fears and threats associated with change. Fear that we may lose some of the mathematics we have traditionally treasured and fear we may no longer be masters of our own domain (our classroom). With these conflicting feelings it is important that we are well informed and clear about the reality of the situation.

I see the graphics calculator as a personal mathematical assistant, a tool at the disposal of students for them to use, as appropriate, in both the learning and application of mathematics. I believe a graphics calculator empowers students, providing them with a toolbox for use in all facets of their learning. Traditionally, our courses have been predominantly algebraic in nature, and while I do not wish to down-play the importance of these skills, they can result in a reliance on this approach to solve problems. By using their graphics calculator, students now also have graphic, numeric and geometric approaches at their disposal when attempting to solve problems. My experience has been that students are becoming more self confident and thus better at solving bigger picture problems as a result of the variety of approaches at their disposal.

I have also noticed an increase in the interaction between students both inside and outside the classroom. It is common to see students gathered around a calculator discussing activities and sharing ideas.

While the graphics calculator has the potential to change the way we do things it is important to consider whether these are in line with the general aims of education. My school has a strategic plan that includes statements relating to:

- preparing students to survive/strive in a technological world

- producing students who are good problem solvers

- producing students who work effectively in groups

- producing life long learners

The effective use of a graphics calculator does contribute positively to meeting these ideals.

Some teachers fear that graphics calculators will remove the mathematical rigour and understanding we have treasured for so long. I do not believe that this is necessarily so, in fact I believe we can only enhance this through the appropriate use of the technology.

In the evolution of graphic calculator use I hope we will move from students saying

'Do I use the graphic calculator for this question?'

to

'One of the methods I tried to solve this task involved using my graphics calculator…'

I believe this will only occur when each student has permanent access to graphics calculator technology. At my school we have mandated graphics calculators at Years 8, 11 and 12 in 2000.

Assessment

When considering the place of graphics calculators in assessment the overriding consideration must be that the assessment program reflects the curriculum as taught and learnt. There is little point in providing a large number of learning experiences through the use of a graphics calculator and then removing them at assessment time. This situation will only confuse and anger students and produce results that do not reflect their learning. Conversely, the situation where graphics calculators are not used in a student's learning but are allowed in assessment leads to a similar mismatch. I believe that a balance between these approaches is needed. There will be particular mathematical skills that we, as teachers, wish to see displayed. The use of statements such as 'show all working' can be used to convey these expectations to the students. Other tasks, without such statements, may be tackled in several different ways.

Adding value to assessment

With the use of graphics calculators there is the potential to gain extra information on our students' learnings. Traditionally, a task with only one feasible method told us whether a student knew and could apply that method. With a graphics calculator, and the potential to use different approaches, the student's level of understanding can be identified. This may have benefits with the implementation of outcomes-based assessment and the need for teachers to designate the level at which a student is operating.

An example may be solving an optimisation task. Traditionally these have involved calculus techniques. With a graphics calculator the lowest level of response would involve using lists to complete simple calculations and then combine these to solve the task. A higher level response would involve some algebraic manipulation of the basic information,

followed by the use of a graphical or numeric technique. The highest level of response may be the ability to solve the task using an algebraic technique (calculus), or even a sensible combination of graphics calculator and calculus approaches.

Master class

This master class focusses on the use of graphics calculators in assessment. The items have all been used at my school as part of the assessment program. These items are all designed with the use of a graphics calculator in mind, although the calculator may not be the main focus of the item. In Queensland, assessment is school-based and enables teachers to design items that reflect the learning experiences of the student from the particular school. Assessment is undertaken in three domains: communication, mathematical techniques and mathematical applications. The last of these is about solving unfamiliar tasks and as such the use of technology as part of the solution process is often very important.

Along with a discussion of the assessment items themselves, student responses to some of these items are also included. The items are intended to prompt consideration of how our understanding of student learning might be improved through these tasks involving graphics calculators. They also raise questions about changes in what we value and what we should/could value in a mathematics curriculum.

Sample assessment items

The following pages are examples of assessment items and student responses. In particular, some of the key questions and dilemmas the samples may help to illuminate are:

- How do graphics calculators add value to our knowledge of student learning through changed assessment practices?

- How do graphics calculators alter the knowledge we value in student learning (as shown by changed assessment practices)?

- Do graphics calculators change what we will/can value in our students learning through the ability to change our assessment practices?

- Does what we can assess drive what we can/do value in our students' learnings?

- What do / should we value in a school maths curriculum?

- Is the rigorous algebraic nature of our maths courses at the end of the 20th century appropriate for our 21st century school?

- Should we place restrictions on what can be done with the graphics calculator?

- What do we expect our students to record when they use a graphics calculator to find a solution?

- Do the gains outweigh the losses?

Sample assessment item 1

Aim: To test student understanding of quadratic functions.

TRADITIONAL APPROACH

Sketch $y = -2(x - 4)^2 + 5$

TECHNOLOGICAL APPROACH

The data below relates to the flight time of whirlybirds with different wing lengths, collected as part of an investigation aimed at creating a whirlybird that stays in flight for the maximum period of time.
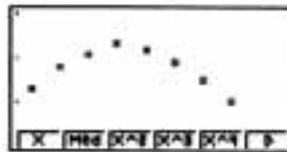
Determine an appropriate quadratic model to fit this data explaining carefully the process used in developing the model

Model is of the form $y = a(x - b)^2 + c$, from the scatter plot a is negative, b is approx 8 and c is approx 2.3.

Try $y = -(x - 8)^2 + 2.3$
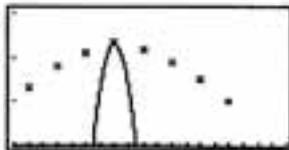
The graph is approximately placed appropriately but the value for a needs to be a smaller negative number.

Try $y = -0.05(x - 8)^2 + 2.3$
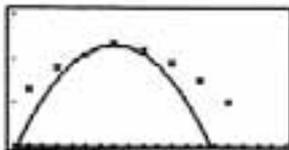
The graph needs to be a little shallower.
Try a = -0.03
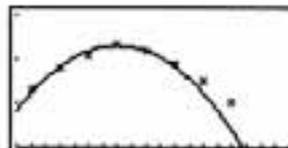
A satisfactory model would be
$y = 0.03(x - 8)^2 + 2.3$

Sample student response 1a

ⓐ
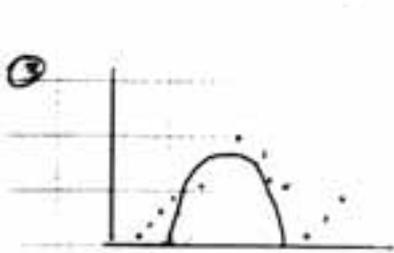
$-0.1(x-8)+2.5$

Good turning point (I think)
still to thin

ⓑ
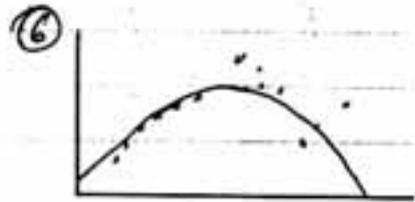
$-0.025(x-8)^2+2.3$
I say good enough ✓

my equation
$-0.025(x-8)^2+23$

after I was happy with my equation
$-0.025(x-8)+2.3$, I wanted to see
what the calculator thought. I went to
STAT → CALC → Quadreg → L_1, L_3 → enters
it gave me

$-0.0239715285 x^2 + 0.4162587413x + 0.5337862138$
I drew the equation and found the TP so
I could convert it into $a(x \pm b)^2 c$
The turning point was:
$x = 8.8673111$ $y = 2.3793343$ which
changes to
$-0.0235(x-8.8673)^2 + 2.3793$

Calc $-0.0235(x-8.8673)^2 + 2.3793$

vs

Me $-0.025(x-8)+2.3$

Sample student response 1b

MODELLING THE DATA.

Initial statement on the basic shape.   *negative*

The plotted points have formed a ^parabolic shape, with a few points being a bit out near the end.

First model: $y = -(x-9)^2 + 2 \cdot 43$

Sketch:

I tried 'a' as −1 first but this was too steep so we need to decrease the value of 'a'.

Comment:

I knew that 'a' would be ⊖ because the basic shape shows a sad parabola (∩). Also, I knew that 'b' would be −9 because it moves the graph 9 → and that 'c' would be 2·43 because it moves the graph 2·43 ↑ · TP = (9, 2·43)

5TH MODEL: $y = -0.04 (x-9)^2 + 2 \cdot 43$.

The model is getting a lot better. It now passes through two points. But we should try and see if this is the best one. To make sure, decrease the value of 'a' once again. Try $-0.03(x-9)^2 + 2·43$.

6TH MODEL: $y = -0.03 (x-9)^2 + 2 \cdot 43$.
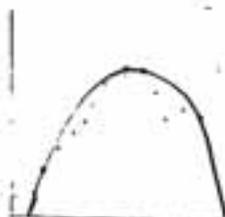
This model fits the points plotted even better. It passes through four points and resembles the basic shape/curvature. the points plotted.

* Therefore my final rule for this model
$-0.03 (x-9)^2 + 2·43$

Sample student response 1c

The sample of student work here shows how the technology can create problems for some students who become fixed on using the technology to generate results which they believe to be more and more accurate. Students such as this have lost sight of the original modelling context and its inherent approximations. The problem requires them to find a graph that will fit the data within the very rough degree of accuracy inherent in the data as gathered. Notice that this student has not generated any graphs to show whether successive attempts are really improving the model. Teachers need to be alert to how some students become captive to the technology and lose sight of the question in hand.

Sample assessment item 2

The Drinking Glass

The aim of this question is to design a drinking glass, using some of the standard mathematical functions that you have studied to define its outline. You only need to design one half of the glass as it is symmetrical and it is easier if the design is done with the glass lying down - ie. with the design running along the x axis.

An example is given below which uses 3 functions- a quadratic for the base, a straight line for the stem and a square root graph for the body of the glass. The tricky part is getting the right co-efficients in your equation so that the bits of your design actually join up. The functions need to have domain statements attached so that only the relevant section of the function is included in the design.

Example:	Wineglass	Function	Domain	Co-ordinates (millimetres)
	Base	$y = -0.35x^2 + 40$	$0 < x < 10$	(0,40) to (10,5)
	Stem	$y = 5$	$10 < x < 60$	(10,5) to (60,5)
	Body	$y = 3.913 \sqrt{x-60} + 5$	$60 < x < 140$	(60,5) to (140,40)

Design Outline

Finished Product (Wineglass)
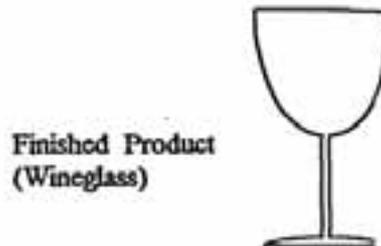
The co-efficients for the function defining the body are derived as follows:

The square root function starts at (60,5) so its model is $y = k(\sqrt{x-60}) + 5$ where the value for k is determined by making the curve pass through the next point (140,40).

Question 1 - A Drinking Glass

My drinking glass comprises of four parts, the top, stem and two parts of the base.

Firstly the top, I wanted to make the top like a martini glass so it had to look like this;

To make this equation I need to draw a straight line graph which goes in the form y=x and this can have numbers added or subtracted from it, as well as number multiplied by x, i.e. y=mx+c. To start with I drew the graph y=x and because I had to move it into positive x values I subtracted a number from x. The window settings at that time meant that the number is 13 and this drew a line like this;
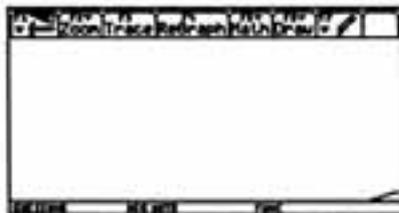
This graph, however, was not steep enough, therefore I need to multiply x by a number, the number I first chose was 1.5 it gave a graph like this;

This graph is perfect.

Now for the stem, I have decided to use a fancy stem since it has to be unique. So I have decided to use a sine curve, the sine curve with 2 added so that it is above the x axis, therefore it is y=sin(x)+2 gives a graph like this;

Sample student response 2a

Page 2
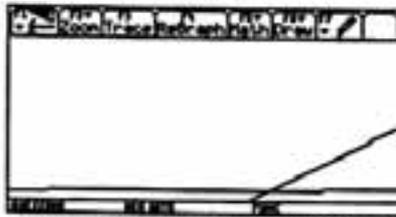
This graph is not what I want, so just to change it to smaller curves, I have changed the angle measure to radians this then gave a graph like this;
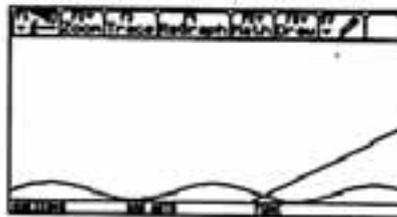
This is now perfect, but the graphs cross paths so I need to make it so they stop where they cut each other. To do this I have gone to the math menu and used the intersection command, this gave an intersection of x=9.77. I then put this in as the starting domain of the first graph and the ending domain of the second graph.

For the first part of the base I have decided to add a knobby bit at the base of the stem, namely a parabola. Parabolas go by the equation y=ax^2+bx+c. Because I want the parabola to be a *sad* face, a has to be negative and it needs to be wider, a=(-3/4) and because it doesn't need to move left or right b=1. Lastly it needs to be up and cut the y-axis at 6 so that it connects with the other graphs, therefore y=6.

This means the equation is, y=(-3/4)x^2 + x + 6.
This gives a graph that looks like this;
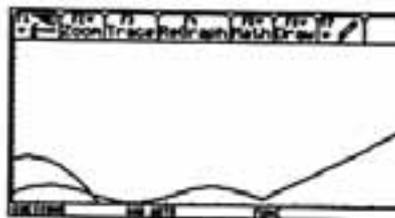
Sample student response 2a

Again we need to work out the point of interception of the two graphs to work out the domains and this is done the same way. This gives an interception point of x=3.0429 and this is put in as the domain for these two graphs.

Lastly is the last part of the base, I have chosen to do another straight line but this time it will be part of an absolute value graph which goes by the equation y=a|b+x|+c. Knowing that the number multiplied by the |b+x| makes the graph thinner or wider and that a whole number makes it thinner I have made the absolute value graph thinner and the number I have used is . The b in the equation moves the graph left or right and I need to move it one place to the right therefore b has to = -1. The graph does not need to be moved up or down and this means that c=0. This makes the equation;

$y=|x+1|$

This gives a graph that looks like this;

The graph below is half of my final design and it looks like;

Sample student response 2a

A sketch of my whole design is;
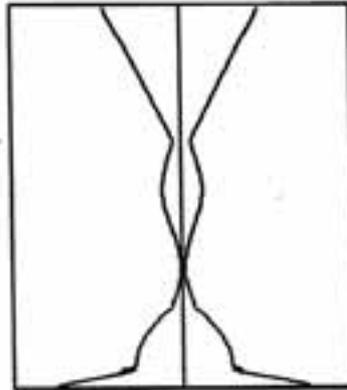
The functions used in this design and their relevant domains are stated below;

PART	FUNCTION	DOMAIN
Top	$Y = 1.5x-13$	$20>x>9.77$
Stem ✳	$Y = \sin(x)+2$	$9.77>x>3.0428$
Top Of Base	$Y = (-3/4)x^2+x+6$	$3.0428>x>0.578$
Bottom Of Base	$Y = 15^*\,\|x-1\|$	$0.578>x>0$

✳ Make sure the calculator is in radians

View window
x min 0
x max 20

Y min 1
Y max 15

Sample student response 2b

Maths B Assignment.

1. Design Outline   Finished Product.

Functions Used.   Domain.

Base :- $y = -0.5 (x+10)(x-10)$ $0 \leq x \leq 10$

Stem :- $y = 1$ $10 \leq x \leq 90$

Bubble :- $y = \sqrt{(100 - (x-100)^2)}$ $90 \leq x \leq 110$

Body :- $y = 5\sqrt{(x-110)}$ $110 \leq x \leq 185$

Base :- For the base I use the function $y = -ax^2 + bx + c$ but converted it to $y = -(x+a)(x-b)$. I chose +10 and -10 because that's where I wanted the curve to cut the x-axis. For the steepness I chose 0.5 because if I left it as $y = -(x+10)(x-10)$ the curve would have cut the y-axis at 100 when I wanted it to cut at 50.

Stem :- Because I want a straight line running across the x-axis I used the function $y = a$. So I could see the line on the graph I used $y = 1$.

Bubble :- The function I used for the bubble was the function for a circle: $(x-H)^2 + (y-K)^2 = R^2$, but I rearranged it. First I sub-ed in the co-ordinates I wanted. $(x-100)^2 + (y-0)^2 = 10^2$. I used those co-ordinates because I wanted the centre to be at $(100,0)$ and the radius 10. I then rearranged it to get $y = \sqrt{(100 - (x-100)^2)}$. (See working to see how).

Body :- I used the function $y = k(\sqrt{x-a}) + b$. Because I wanted the line to curve at $y = 0$ I didn't need a "b" value. "k" = steepness so I changed the value until I found a value that I thought would suit. I chose 5. "a" = 110 because that's where I wanted the curve to cut the x-axis.

Sample assessment item 3

(simple)

The temperature at the top of Mount Lady Mary (on the moon) in degrees celcius is given by the function

$$L(t) = -175\cos 0.23t - 75$$ where t is the time in days after sunrise.

The temperature at Oremus Carter is given by the function

$$O(t) = 172\sin 0.23t - 80$$ where t is the number of days after sunrise at Mount Lady Mary.

Your task is to determine a rule that gives the difference in temperature between the two places at any time t. (Your answer must be in terms of cos or sin).

(simple)

Times of sunrise and sunset vary during the year. The table below shows times for Sydney for every two weeks of the year, starting in the second week of January. (Times are Eastern Standard Times and have been converted into decimal hours)

Week	Sunrise	Sunset	Week	Sunrise	Sunset
2	4.90	19.15	28	7.00	17.02
4	5.13	19.10	30	6.88	17.17
6	5.37	18.95	32	6.70	17.33
8	5.58	18.71	34	6.45	17.48
10	5.78	18.43	36	6.15	17.65
12	5.96	18.12	38	5.83	17.80
14	6.15	17.80	40	5.52	17.97
16	6.32	17.50	42	5.20	18.15
18	6.50	17.25	44	4.95	18.35
20	6.67	17.05	46	4.75	18.57
22	6.83	16.92	48	4.63	18.80
24	6.95	16.88	50	4.63	18.98
26	7.02	16.92	52	4.72	19.12

Produce a model for the number of hours of daylight at any time during the year and estimate the total number of hours of daylight during the Olympics.

Sample student response 3

CIII – Mathematical Applications (Simple)
Question 1

To find the difference between temperatures it is simply a matter of subtracting one from the other, or in this case, one graph from the other.
When the original graphs are put into the calculator they look like;
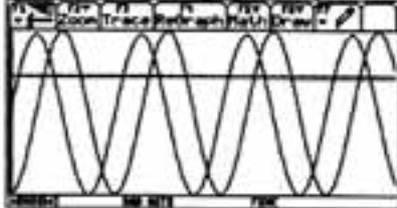
The graph that starts greater is O(t) so I have chosen to take L(t) away from it.
To do this I simply put the equation;
$Y = (172 \sin (0.23t) - 80) - (-175 \cos (0.23t) - 75)$
This produced the graph;
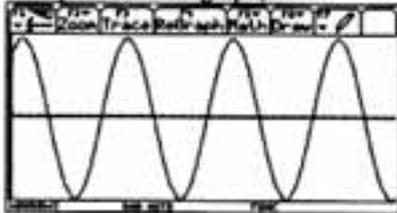
It was then simply a matter of working out the equation.
Firstly we can say it's a Cos curve because it starts up high.
The basic for of a Cos curve is;
$Y = A \cos(B (x + C)) + D$

$A = (\text{Max Value} - \text{Min Value})/2$
$= (240.5 - {}^-250.4)/2$
$= 245.4$

$B = 2\pi/\text{period}$ (because period $= 2\pi/B$
$B = 2\pi/27.3$

$C = 3.377$ because the first max is 3.377 days from the Y axis

$D = -5$ because the graph has moved down 5 places

So therefore the equation to find the difference in temperature between Mount Lady Mary and Oremus Carter is;
$Y = 245.4 \cos (2\pi/27.3(x - 3.337)) - 5$

Sample student response 3

Question 2

For this question, we are asked to find an equation for the number of hours of daylight at any time during the year. We are given the sunrise and sunset times for every two weeks throughout the year.

There are two ways of going about this, one is to find and equation for the sunrise times and one for the sunset times and work out the equation for the difference. The second is to work out the number of hours of daylight for each section and work out an equation from that. I have chosen to do it the second way as it is easier.

First the number of hours of daylight needed to be worked out (in decimal hours);

Week	Hrs Of Daylight	Week	Hrs Of Daylight
2	14.25	28	10.02
4	13.97	30	10.29
6	13.58	32	10.63
8	13.13	34	11.03
10	12.65	36	11.5
12	12.16	38	11.97
14	11.65	40	12.45
16	11.18	42	12.95
18	10.75	44	13.4
20	10.38	46	13.82
22	10.09	48	14.17
24	9.93	50	14.35
26	9.9	52	14.4

Plugging this into the calculator, it gives a scatter plot that looks like;

From this then we can work out an equation.

The first thing that can be said is that it is a Cos curve because of where it started.

So the equation will be in the form;
$Y = A \cos (B (x + C)) + D$

A will again be (max – Min)/2
$A = (14.4 - 9.9)/2$
$A = 2.2$

$B = 2\pi/$ period because period $= 2\pi/B$
$B = 2\pi/54$

Sample assessment item 4

In an ironman event Grant Penny is 1.5 km off shore. He aims to break the record of 1 hour and 6 minutes for this event. Determine how far up the beach he should head to minimise the time taken to complete the race given that he can travel at 8 km/hr on the water and at 10 km/hr on the land and hence state by how much he either breaks or fails to break the record by.
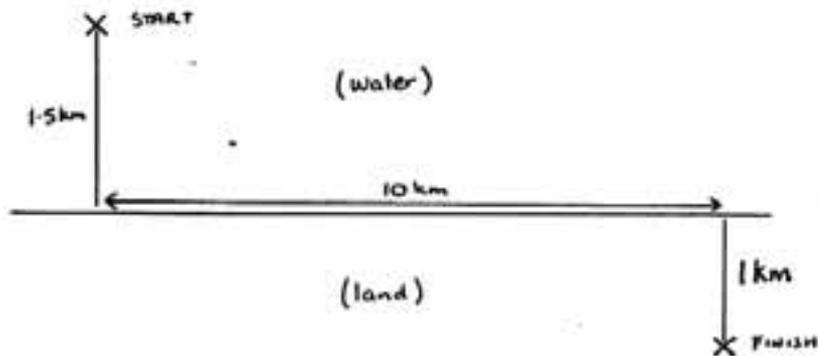
Sample assessment item 5

A company using gamma radiation sources uses steel walls to shield the sources. They wish to replace the steel walls with lead walls because they can be thinner. The present steel wall is 18 mm thick. The new lead wall is to transmit the same percentage of gamma radiation as the steel wall.

The relationship between the Transmission of gamma radiation and the thickness of the wall is known to be an exponential decay model of the form $T = Pe^{-kt}$, (or $T = Pb^t$), where T is the transmission score of the gamma radiation, P is the gamma radiation score without a wall, k is the constant (which depends on the material) and t is the thickness of the wall.

Data collected on the two substances is given below.

Lead

wall thickness	0	2	4	6	8	10	12
transmission score	4160	3760	3200	2820	2490	2080	1850

Steel

wall thickness	0	2	5	7	10	24
transmission score	4160	4090	3830	3790	3460	2820

How thick should the lead wall be?

Sample assessment item 6

Yöu are the inventor of a coffee-making machine as shown below.
Cold water is added to the tank marked A. This water boils and pours out through B into the
funnel at C and into the pot at D. This is kept warm by boiling water still in the tank.
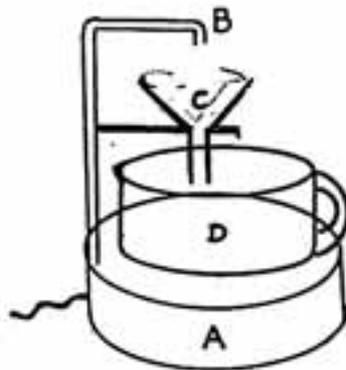
A paper filter in the shape of an inverted cone is used to hold ground coffee beans in the
funnel.

TASK 1 (Completed in small groups in class)

(i)   Cut out the shape on the attached sheet.
(ii)   Fold it into a cone shape, using a paper clip to hold it in place.
(iii)   Calculate the volume of the cone.
(iv)   Alter the cone and compare the volumes of different cones.

TASK 2 (Individual investigation and report)

If your coffee making machine is to operate at maximum efficiency the shape of the funnel
and filter paper will need to hold maximum volume of coffee. Investigate various cone shapes
and discuss the dimensions of the funnel and filter you would use. Explain how you formed
this conclusion. Justify your choice of the dimensions of the shape chosen.
(All calculations, diagrams, graphs should be included)

Sample assessment item 7

Below is the cross section of a hill into which the local council wishes to build a 20 m wide roadway with a gradient of 1/8. Assume this cross section holds for the 20 m depth required for the road.

Your task is to model the cross section of the hill, mark in the new road and determine where and how much excavation and filling must occur to complete the task.
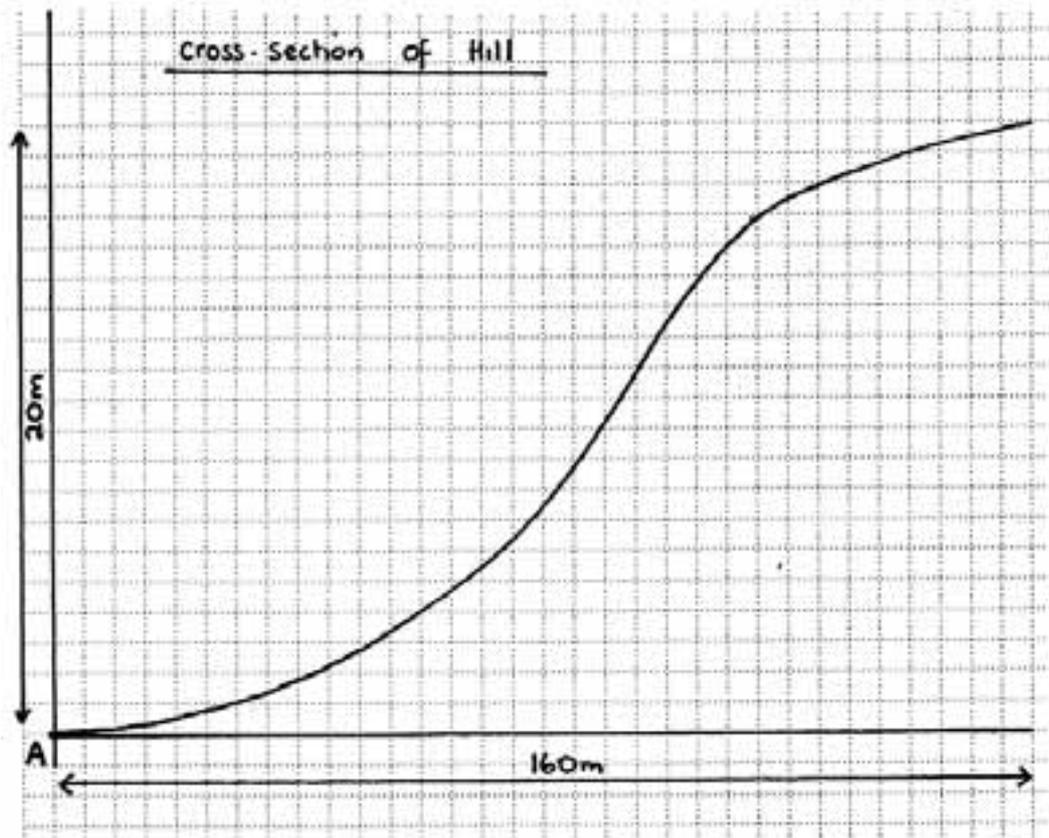
Summary and Reflections

Peter Hickey

Toupe High Languages School, New South Wales

Gary introduced his master class with the analogy that VAT (Value-Adding Technology) was like the GST (Goods and Services Tax) in that it was new, there are many rumours surrounding it and that it comes with its associated threats. He went on to demonstrate how graphing calculators empower students in their learning of mathematics. Along the way he showed that graphing calculators are good for…

- organising data

- removing the pressure of symbolic manipulation

- allowing the student to focus on the process

He stated that the graphic calculator is a personal assistant available to the student in their mathematical journey.

In Gary's school students in Year 8 and Year 11 are required to have their own graphing calculator. Students have been keen in the uptake of the technology. He has witnessed students sitting together in the playground discussing and experimenting with them.

Gary's decision to require students to purchase their graphing calculator was based on his school's aims for education…

- communication

- problem solving

- working in groups

- using technology.

Gary was adamant that assessment practice should reflect learning experiences. If we are going to use the technology in the learning process then we should also assess with it. If we are not going to allow calculators in examinations then why use them at all?

Gary posed a dozen or so questions/dilemmas for the participants to consider which I have summed up as follows:

Is the rigorous algebraic nature of our maths courses at the end of the 20th century appropriate for our 21st century school?

In his efforts to shed some light on these questions/dilemmas, Gary's master class highlighted examples of students work assignments/investigations on:

- the relationship between wing length and time of flight of whirlybirds

- the drinking glass

- temperatures on mountains and craters of the moon

- gamma radiation yields.

Graphics Calculators and Assessment

Mike Ristovsky

Christ Church Grammar School, Western Australia

What are the policies and practice in place at Christ Church Grammar School?

Graphic Calculators have been part of the external Tertiary Entrance Courses throughout Western Australia since 1998.

Graphic Calculators have been part of Christ Church since 1996. All Year 10, 11 and 12 own their own. Years 8 and 9 have class sets, with overhead panels.

My role as second in charge is to provide staff and students with the skills to effectively and efficiently use the technology. This includes developing handouts, assignments, tests and investigations with a technological flavour, without being excessive.

We see a need to develop three types of questions:

- graphic calculators are useless (e.g. probability questions);

- any calculator will be sufficient (usually relate to the type of work students have been dealing with in class);

- graphic calculators are essential (e.g. solving 4 by 4 systems of equations).

What projects and assignments do we have in place?

Several examples are included:

- Co-ordinates and Shapes

- Mortgages: A look at repaying home loans using the financial feature of the graphic calculator

- Creating a Slide Show.

Co-ordinates and shapes

This example illustrates integration of assessment and learning, and the need to support students to use their calculators effectively. Students first learn how to use the 'DRAW' functions to draw and store quadrilaterals and various other shapes. In order to work with and develop appreciation of 'co-ordinates' we will use the commands from the home screen rather than use the pen facility of some calculators.

1. Begin by having a sheet of graph paper to work on in conjunction with the graphics calculator. Draw a set of axes on the graph paper according to your 'WlNDOW' settings. Clear all current graphs and drawings from the calculator. Using the 'WINDOW' button set our axes as seen in Figure 1.
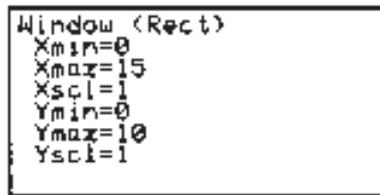
Figure 1

2. From the home screen activate the 'DRAW' function (above the 'STAT' function) and choose option A2. To draw a line from (1,6) to (1,1) we complete the instruction as **Line(1,6,1,1)**, and press 'ENTER' (see Figures 2 and 3).
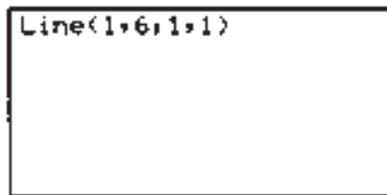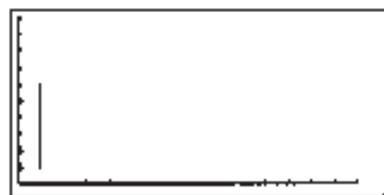
Figure 2   Figure 3

3. To return to the home screen and edit the co-ordinates we press 'QUIT', and then 'ENTRY'. We wanted to draw a horizontal line from (1,1) to (9,1) and so we change the instruction from **Line(1,6,1,1)** to **Line(1,1,9,1)**. After doing this we press 'ENTER' to view the graph (see Figures 4 and 5).
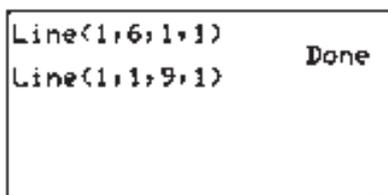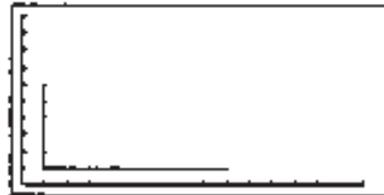
Figure 4   Figure 5

Students could then identify the third vertex to complete the rectangle.

They are then given a worksheet containing the screens below (Figure 6). They are asked to name the quadrilateral and reproduce them on their calculators.
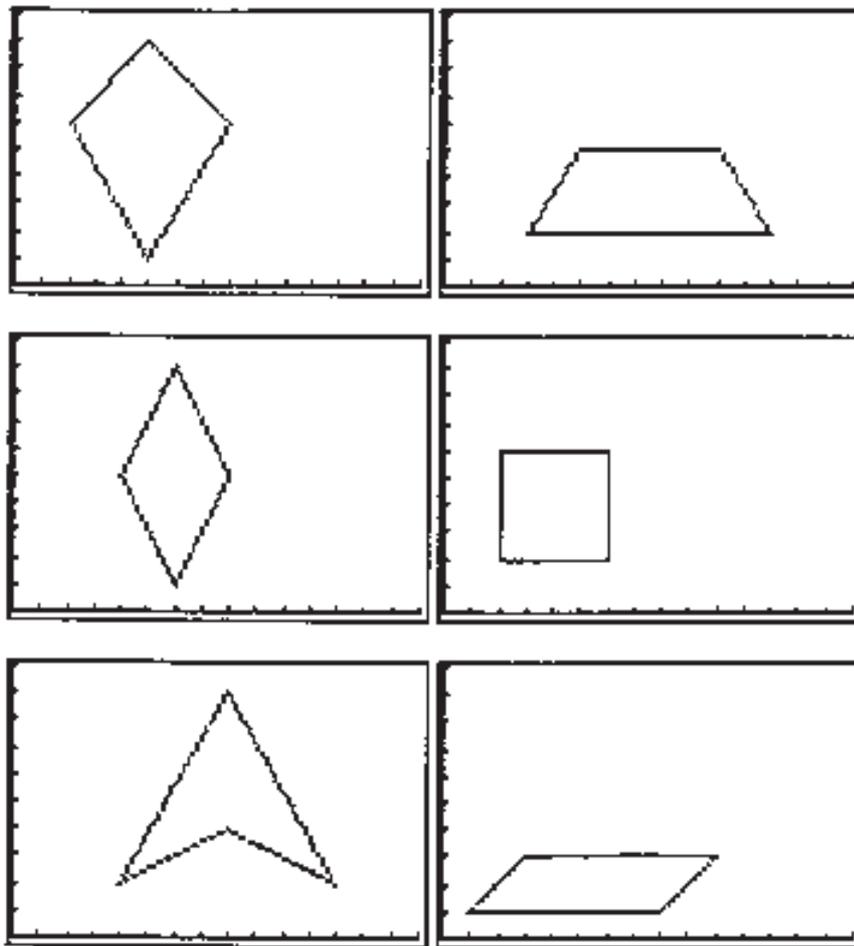
Figure 6. Figures on student worksheet.

A problem — making mistakes

Some of the students will discover that if they entered an incorrect co-ordinate in a 'Line' instruction and then pressed 'ENTER', they could not delete the incorrect number. They have to start all over again!

In Figures 7 and 8 a number 9 has been entered by mistake in the final line, which should have read Line (9,6,1,6).

If you enter a coordinate incorrectly, you will have to redo each Line instruction in order to construct the correct rectangle.
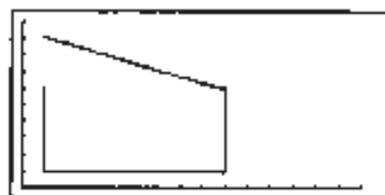
Figure 7   Figure 8

There are two ways around the problem of incorrectly entered coordinates.

1. You could enter the co-ordinates as follows: **Line(1,6,1,1):Line(1,1,9,1)**, using a colon between each line instruction, the co-ordinates can then be checked (and edited if necessary prior to pressing 'ENTER' (see Figures 9 and 10). The disadvantage of this method is that the shape cannot be seen until the end.
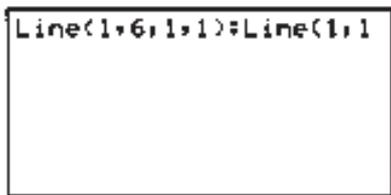
Figure 9   Figure 10

2. An alternate method which proves much more flexible is to write a simple program incorporating a set of Line instructions. If an incorrect co-ordinate is entered the program can be edited without having to key in again the whole set of line commands. An example of such a program can be seen in Figure 11.
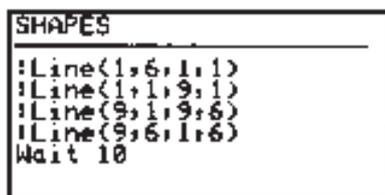
Figure 11

Further assessment

Students are asked to draw any interesting pictures using coordinates and quadrilaterals (examples in Figures 12 and 13).
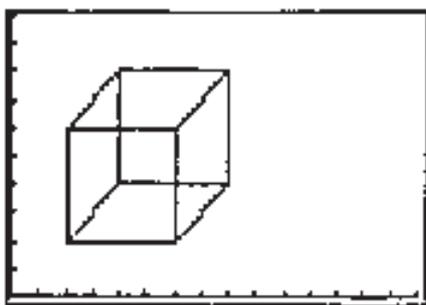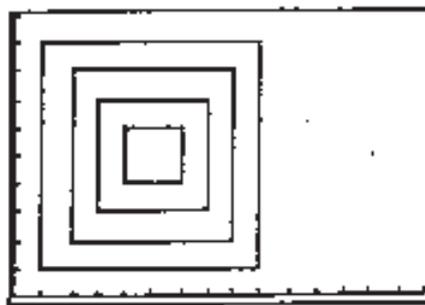
Figure 12   Figure 13

Extension

As an extension to this exercise students can then be encouraged to work both from the home screen (using other draw options) and using the pen facility to produce some creative artwork, which can be saved and printed at a later date (see Figure 14).

Saving a picture

Once you have your picture complete simply select option F1 under the 'DRAW' button and store the picture as a number from 1 to 9. Option F2 will allow you to recall the stored picture.
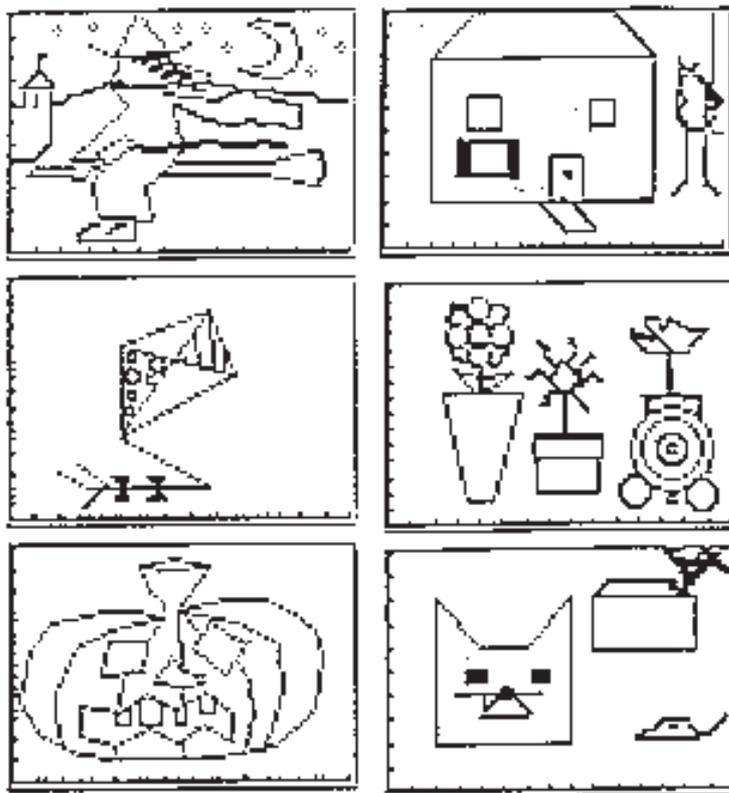
Figure 14. Some examples of student work.

Year 10 investigation: Mortgages

Note: This example again integrates learning and assessment. Students first explore some of the built-in financial features of their calculators, then use this to solve some problems with a view to having their skills and understanding assessed through a 'normal' test.

This investigation will require you to use the financial function of your graphics calculator. By working through a few questions you should become familiar with the options of this feature. You will be asked similar questions under test conditions as a validation of this investigation. Show any necessary calculations as displayed in part (c) of the solution to the example.

For example: Bruno borrows $250 000 for a new home in West Perth and wishes to pay the loan back over 30 years (the maximum term). The interest rate is at 6.5%. He wishes to make his repayments in monthly installments.

a)   What are the monthly repayments?

b)   How much will Bruno repay over the 30 years (include the initial $250 000)?

c)   How much of each monthly repayment is interest?

d)   If Bruno wishes to repay $2000 per month, how much time will this cut off the loan?

In Figure 15:

N = time period (30 years by 12 months per year, hence 360)

I% = interest rate

PV = present value of the loan

PMT = Amount to be paid per time period (monthly), yet to be determined.

FV = future value of the loan

P/Y = number of payments per year

C/Y = number of times compounded per year (same as P/Y)

```
Payment:END
 N=360
 I%=6.5
 PV=250000
*PMT=-1580.170059
 FV=0
 P/Y=12
 C/Y=12
```

Figure 15

Once you have entered all the known information, move the flashing cursor over the unknown (PMT in this example). Now press the 'EXE' button to execute an answer. The negative represents monies leaving your pocket to pay off the loan.

Solutions:

a)   Monthly repayments are $1580.17

b)   Bruno repays 360 lots of $1580.17 = $568,861.20

c)   The interest paid per month will be $\dfrac{\left(568861.20 - 250000\right)}{360} = \$885.73$

d)   In Figure 16 we now need to set all the known information, leaving the 'N =' line to be evaluated. When all the data is entered, including the NEGATIVE $2000, move the flashing cursor to the 'N=' section and press the 'EXE' button.

```
Payment:END
*N=209.2467
 I%=6.5
 PV=250000
 PMT=-2000
 FV=0
 P/Y=12
 C/Y=12
```

Figure 16

The answer of 209.2467 represents months, so divide this by 12 to obtain 17.437 years to pay off the loan. Bruno has effectively slashed 12.56 years off his loan by paying $2000 per rather than the minimum of $1580.17.

Questions

1. Bruno's brother Mario wants to purchase a new speed boat worth $30 000. He takes out a personal loan for the whole amount at 11.5% to be paid over 10 years in monthly repayments.

    a)   What are the monthly repayments?

    b)   How much will Mario pay back over the 10 years?

    c)   How much interest is paid per month?

    d)   How soon could he pay the loan off if he paid 1.5 times the minimum repayments per month?

    Mario's other brother Dino discovers that at the bank down the road they are offering 10.5% on personal loans. Redo your calculations in question 1 and answer (a) to (d) if he decides to go with the 10.5% deal.

2. Dino's sister Maria decides it is time to purchase her first car after completing her schooling. She borrows $8000 from the bank over a period of 5 years at an interest rate of 10%. She decides to repay the bank in fortnightly repayments.

    a)   What are her fortnightly repayments?

    b)   How much will she repay over the 5 years?

    c)   How much interest is paid per fortnight?

    d)   How much will she have to pay per fortnight if she wants to repay the loan in 3 years?

    e)   By how much will her minimum repayments increase if the bank moves its rate from 10% to 12.5%?

    f)   How much is she saving over 5 years by paying fortnightly rather than monthly with the interest rate at 10%?

    Maria's sister Gina decides to go to the same bank with all the same terms to borrow $15, 000 for a sporty car. Her repayments are fortnightly too. Work out (a) to (f) above for her deal.

3. The father of these children (Patrick) decides to build a bigger house to accommodate them all. He borrows $300 000 from the bank at 7% and wishes to make monthly repayments over 30 years.

    a)   What are the minimum repayments?

    b)   How much will he repay over the 30 years?

    c)   How much interest will he repay per month?

    d)   If he pays $2500 per month, how long will it take him to repay the loan?

    e)   If he wishes to repay the loan in 25 years, how much will he have to repay per month?

f)   If the bank lifts its interest rate to 8.5%, over the 30 years, by how much will the minimum repayments from (a) increase?

g)   Refer back to the initial question. If he repays the loan weekly (52weeks = 1 year), rather than monthly how much money will be saved over the 30 years? How much does this equate to each year?

h)   Repeat this question (3) parts (a) to (g) above for a loan of:

    i) $100 000

    ii) $200 000

Creating a slide show on the EL-9600

Creating a slide show on the EL-9600 is a simple procedure that can be used as an assignment or a consolidation lesson whereby students can come to the front of the class and present their slide show. It represents a different means of presenting their work for assessment. All that is required is for students to produce their own slide show and transfer it to the OHP-equipped calculator to be displayed on the white board.

The short example that follows assumes that the students have completed work on finding the zeros of a quadratic. It is now up to them to present a slide show to demonstrate their learning.

1.   After your calculator has been reset (which is not absolutely necessary) activate the 'SLIDE SHOW' button and choose option C. You should be prompted to enter the TITLE. The alpha lock has been automatically switched on, so simply type in a title using the blue keys and press enter. The example will be titled 'Zeros' (see Figures 17, 18 and 19).
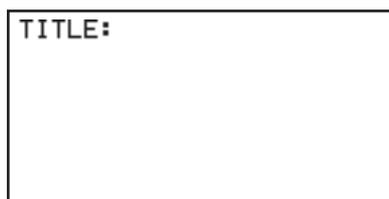
Figure 17   Figure 18
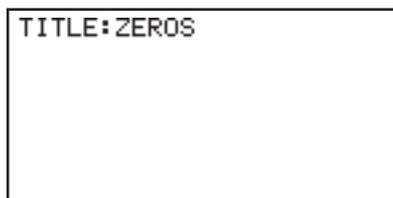
Figure 19

2.   Activate the 'Y=' button and enter $x^2 + 2x - 3$. Once you have entered the equation activate the yellow 'CLIP' button above the 'SLIDE SHOW' button. The equation you have entered will become the first screen in your slide show. (You should be able to

see the message 'Storing screen 1' briefly appear on your screen). Now press the graph button to view the graph. Once you can see the graph activate the 'CLIP' button again. This becomes screen 2 of your slide show. To find the roots or zeros of this equation activate the yellow 'CALC' button and scroll to option 5. Before pressing 'ENTER', 'CLIP' this as your 3rd screen. Once you have pressed 'ENTER' and can view a zero, 'CLIP' it as your 4th screen. Repeat this procedure and 'CLIP' the other zero as your 5th screen (see Figures 20 to 24 for the complete slide show).
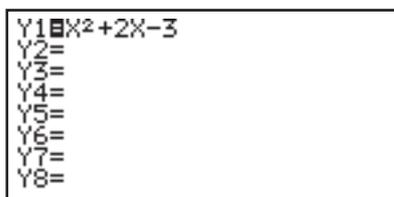
Figure 20. Enter your quadratic
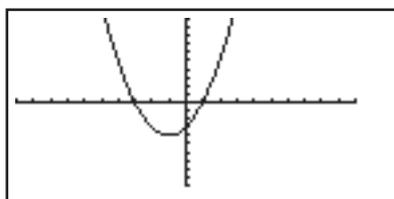
Figure 21. View the graph of your quadratic

Figure 22. Select A5 from 'CALC'.
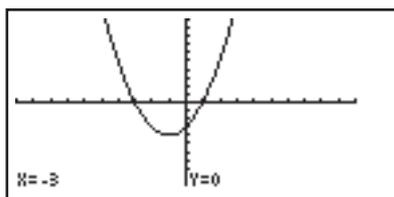
Figure 23. 'CLIP' an intercept.
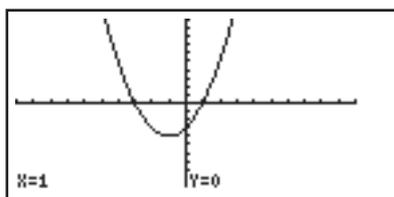
Figure 24. Repeat procedure in Figure 6 to view the other intercept.

3. Now you have completed your show it is time to give it a test run. Press the 'SLIDE SHOW' button and activate option B by pressing enter once 'B ORG' is highlighted. Keep pressing ENTER to view your slide show. Use the cursor key to scroll backwards through the slides.
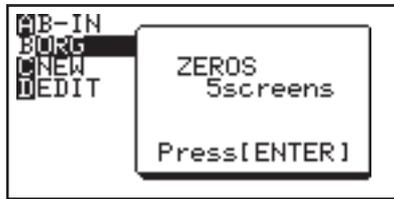
Figure 25
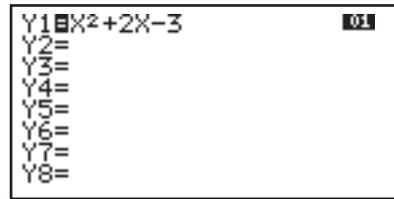
Figure 26
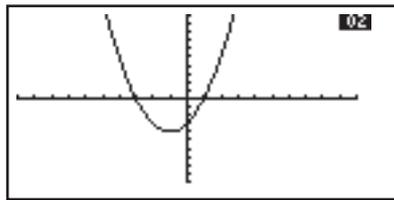
Figure 27
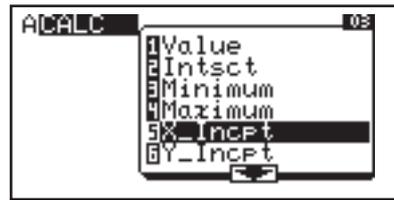
Figure 28
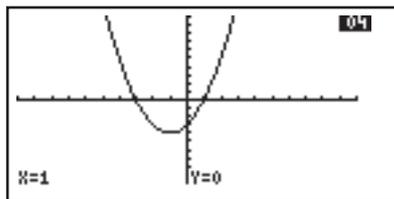
Figure 29
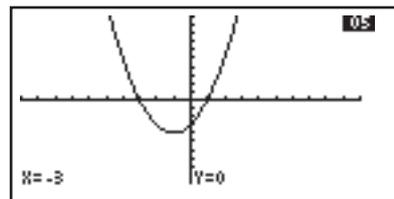
Figure 30

Having the equation on the screen when viewing the quadratic (option C1 under the 'FORMAT' button), and various other features, will enhance your slide show.

4. To transfer the data from your calculator to the OHP-equipped calculator, join the two with the connection cable. From the home screen on your calculator, choose the yellow 'OPTION' button and select option D1 and press enter. Now scroll to option A10 and press enter. Once your slide show is highlighted press enter and an asterix will appear near the title. On the OHP calculator choose D2 under the 'OPTION' button to receive. Now simply press the 'EXE' button on your calculator to execute the command.

Your slide show is now on the OHP calculator and ready for your presentation (see Figures 31 to 34).
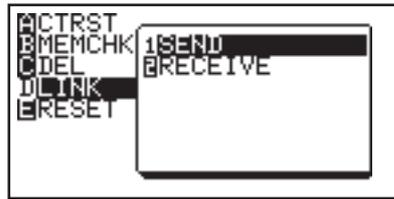
Figure 31. Send from your calculator.
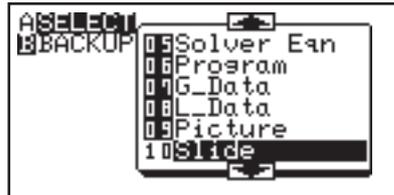
Figure 32. Select the slide show.
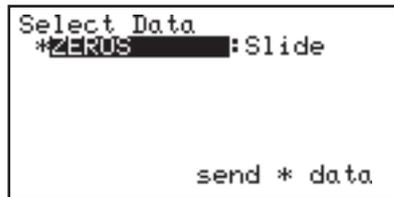
Figure 33. Asterix which to send
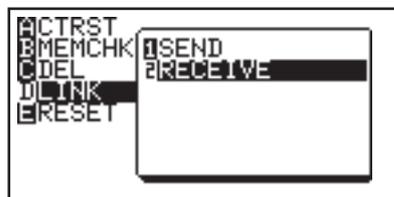
Figure 34. Receive from OHP calculator. Press 'EXE' to execute from your calculator.

Appendix — some examination questions

Christ Church Grammar School Year 10 Final Examination (Section C) 1999

You should show all working in this section. Marks will not be awarded for answers without supporting evidence.

1. [3 marks]

   An investor buys a painting for $2800. He estimates that it will increase in value by 9.5% per annum (compound interest). After how many years (correct to 2 decimal places) will the painting be worth $8000?

2. [4 marks]

   Tennis balls have diameters of 9 cm. They are packed in cylindrical metal tubes which are sealed top and bottom. The tennis balls just touch the sides and the top and bottom of the tubes. Find the volume of empty space (correct to 2 decimal places) in a tube containing five tennis balls.

18. [8 marks: 4, 1, 3]

    (a) Solve simultaneously:

    $a + b + c + d = 0$
    $a + c + d + e = 5$
    $a + b + d + e = 1$
    $a + b + c + e = 2$
    $b + c + d + e = 4$

    (b) In Alcatraz, a prisoner waits on death row. By custom, the night before a man is to be executed, he plays a game — either of chance or skill, it is the judge's discretion. This game will decide whether the prisoner will indeed die, or be set free. This particular prisoner was presented with a game that was perhaps a little of both. Before him are two large urns. One urn contains fifty black balls, the other fifty white balls. Tomorrow, the executioner will, while blind-folded, draw a ball randomly from one of the two urns. If it is black, the prisoner dies. If it is white, he will be set free. If the urn is empty he draws again from the other urn. The prisoner wants very much to be set free, and is pleased that with the current state of affairs, his chances of being set free are fifty-fifty. He is then presented with an option — he may change the contents of the urns. He can swap white balls for black, move balls from urn to urn, etc. There is a stipulation that when he is done, there must be fifty white and fifty black balls total between the two urns. It occurs to the prisoner he might be able to help his situation by moving the balls so that there were twenty-five of each colour in each urn, then making sure the white balls were on top. But the executioner might have guessed this, and may shake up the urns. Worse yet, he might deliberately reach to the bottom of the urn he chooses.

    i. Is there another way the prisoner can help himself?

    ii. If so, what is the probability he will be set free? If not, why not?

Curriculum Council (1998) Applicable Mathematics TEE

(b)   Use your graphics calculator to help answers this question.

(b)   In the construction of a building, aggregate (a mixture of small stones) is required for concrete mixing. The quantities w, x, y and z (in cubic metres) are to be obtained from four different locations. The following system of equations was obtained from considerations of the total quantity required, transportation costs, quality of aggregate and availability.

$w + x + y + z = 10\ 000$
$0.02w + 0.015x + 0.01y + 0.025z = 150$
$0.015w + 0.02x + 0.015y + 0.005z = 155$
$-w - 4x + 3y + 2z = 0$

i.   EXPLAIN how the solution to the system of equations can be obtained using an inverse matrix.

[2]

ii.   Write down the appropriate inverse matrix.

[2]

iii. Write down the solution to the system of equations.

[2]

(7 marks)

The probability density function for a continuous random variable X is given by

$$f(x) = \begin{cases} \dfrac{x(x+2)(x-6)(x-7)}{6.552} & 0 \le x \le 6 \\ 0 & \text{elsewhere} \end{cases}$$

(a)   Find the following probabilities.
    i. $P(X < 2.6)$ [2]
    ii. $P(X > 3.2)$ [2]
(b)   Is the distribution symmetric? Justify your answer.   [3]
(7 marks)

Solve the following inequality correct to 2 decimal places.
$(x^2 - 1)(x^2 - 2x - 15) \le 0.75 + 3x - x^2$
Explain briefly how you arrived at your answer

15.   (10 marks)
$x + 3y - z = 5$
$2x + 7y - 3z = 6$
$-x + 4y + (p^2 - p - 6)z = p - 33$
For what values of p will the above system of equations have
i.   an infinite number of solutions?
ii.   no solutions?
iii. a unique solution? Find this solution in terms of p.

Summary and Reflections

David Leigh-Lancaster

Board of Studies, Victoria

The session led by Mike was based on a paper *Graphics Calculators and Assessment* used in a school faculty context. This paper outlined several examples of how graphics calculator technology had been used to develop innovations in the context of school based mathematics assessment at middle secondary and upper secondary levels. The latter was also linked to the expected use of graphics calculators in the Curriculum Council external examinations in Western Australia.

Contexts for these assessment episodes included:

- a project and problem related to the use of point plotting, line and shape drawing to explore the relationship between coordinate specifications of location and geometric shapes constructed using points, lines and curves (Year 8);

- a practical investigation involving the use of built-in TVM (Time - Value - Money) module as a tool to explore a loan scenario, in particular the behaviour of 'the loan' in terms of variation of the parameters involved (Year 10);

- an assignment involving the creation of a slide show to review and consolidate work on the analysis of quadratic functions and related graphs as a class presentation;

- consideration of graphic calculator 'active' examination questions from both external (Year 12) and school based (Years 10 and 11) examinations.

The term 'innovate' derives from the Latin prefix 'in-' (towards) and the verb 'novare' (to make new). Throughout history the emergence of new technologies has been a powerful stimulus in moving people towards developing new approaches for their various activities. In mathematics such technologies have often been related to improving the efficiency of computation, the most recent example being the development of the scientific calculator.

The relatively recent emergence, over the last decade, of affordable graphics calculators which enable the user to graph functions and relations, produce tables of values, manipulate lists and matrices, obtain summary statistics, solve equations and systems of equations numerically and find numerical derivatives and definite integrals, has made a comprehensive and powerful tool accessible to teachers and students alike.

This technological innovation has led to innovation in approaches to teaching and learning of mathematics content from existing curricula. Examples of this include curve sketching, a less artificial or restricted treatment of some areas of study such as matrix algebra and statistics and the development of new areas of study such as the numerical and graphical analysis of differential equations which do not have readily known analytic solutions. The link to innovation in assessment flows from the close and vital links between teaching, learning and assessment.

Discussion arising from this session identified several key issues and principles for assess-

ment in mathematics where graphics calculator technology is involved (these are not listed in any particular order) are:

1. Innovation in assessment is context dependent and may involve new approaches to existing content and material, or may provide the opportunity for new assessment items to be developed (for example, the solution of higher order systems of simultaneous linear equations).

2. Where graphics calculator technology is used in the teaching and learning of mathematics, it should be available for use in at least some of the related assessment.

3. There are strong links between what is 'valued' and what is 'assessed' — if the appropriate and effective use of technology in mathematics is to be encouraged this cannot occur in isolation from assessment that involves such use.

4. Clear systemic directions are needed with regard to the relation between the use of graphics calculator technology in assessment (not allowed/permitted/ assumed/expected). Expectations with respect to some or all components of formal systemic assessment requirements in both examinations and school based assessment need to be clearly articulated, and supported by the publication of sample assessment items.

5. If innovative practice in assessment is to become accessible to a broad range of teachers, it will require effective support from high quality professional development and supporting resources.

6. The dissemination of innovative assessment practice, and the opportunity for discourse on such practice, requires the coordinated involvement of subject associations, systemic authorities and professional associations.

7. The effectiveness of innovation needs to be carefully monitored and supported by subsequent research.

8. Technology should not be used to 'drive' assessment practice, specific graphic calculator capabilities should not necessarily be used in assessment merely because they are 'there', nor should the complexity of problems be arbitrarily increased because of the capabilities of graphics calculators.

As different approaches to assessment involving graphics calculator technology are trialled, there will be an increasing awareness of the potential and pitfalls associated with such approaches. Robust and critical discussion of these innovations will be vital if they are to inform further developments in effective assessment of student learning in mathematics.

Teaching Mathematics in Context through the Use of Technology: the Mathematics of Bézier Curves

Jon Roberts

Loreto College, South Australia

Bézier curves are used by computer aided designers to draw what would have been free-hand smooth curves if the design were done by hand. They bring forward a geometrical conception of cubic splines that allows the CAD worker to draw and shape the curves intuitively and interactively. The meshing of vectorial linear combinations with parametric curves through the use of matrices or polynomial regression, and the subsequent analyses of such curves through the use of parametric calculus provides powerful insights for today's students into the role and relevance of mathematics in today's technological world.
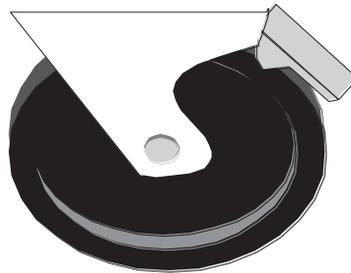
Figure 1. A castor wheel drawn with Bézier curves in *Corel Presentations*.

What is A Bézier curve?

- A smooth curve used in computer aided design that can be edited 'intuitively' by the designer to meet aesthetic requirements (see Figure 1).

- A contemporary context, accessible to anyone using a recent computer drawing package such as those included in the integrated 'Office suites' offered by many software companies.

- A mathematically rich context which can be explored geometrically by students from Year 7 upward using the interactive geometry programs available on computers and graphics calculators (see Figure 2).

- A curve described by parametric equations, which, in design applications, take the form of polynomial equations up to degree three (see Figure 3).

Figure 2. Letters designed by a Year 7 student using Bézier curves in *Cabri Geometry II*.
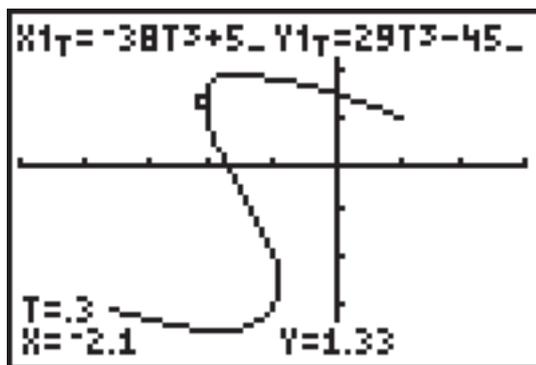
Figure 3. A Bézier curve drawn from its parametric equations using a TI-83 calculator.

Investigating Bézier curves using a graphics calculator

Originally implemented only in expensive CAD packages, Bézier curves are now much more widely available and, as can be seen from the examples above, can now be constructed and studied geometrically, algebraically and numerically. Janke (1993) discusses the application of Bézier curves in high-end computer design packages. Since then, technological developments have made the study of such curves even more accessible. Roberts (1999) proposes a geometrical and numerical investigation of these curves, suitable for senior secondary students, using interactive geometry and the polynomial regression features available on modern graphics calculators.
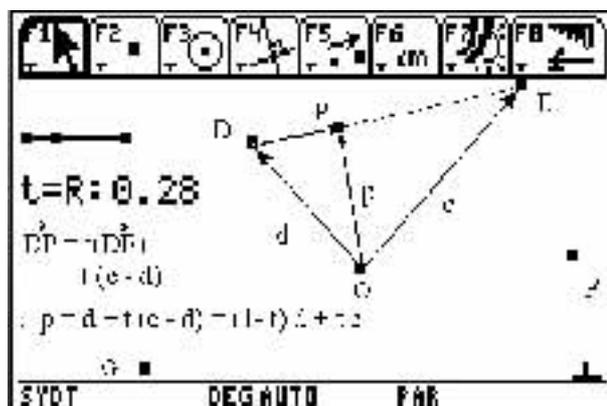
Figure 4. Investigating the construction of linear combinations of vectors using interactive geometry features of the TI-89.

Initially implemented as computer software, interactive geometry has, in recent years found its way into graphics calculators, and is now one part of the features of at least three hand-held devices. Incorporation of interactive geometry and symbolic algebra with the standard graphics calculator features provides students with a powerful and motivating tool for the study of mathematics in context.

The geometry of Bézier curves

Geometrically the Bézier curve is constructed as a set of nested convex linear combinations. Through the use of macro constructions students are able to explore the properties of linear combinations, and the effect of iterating them to construct the Bézier curve as a locus.
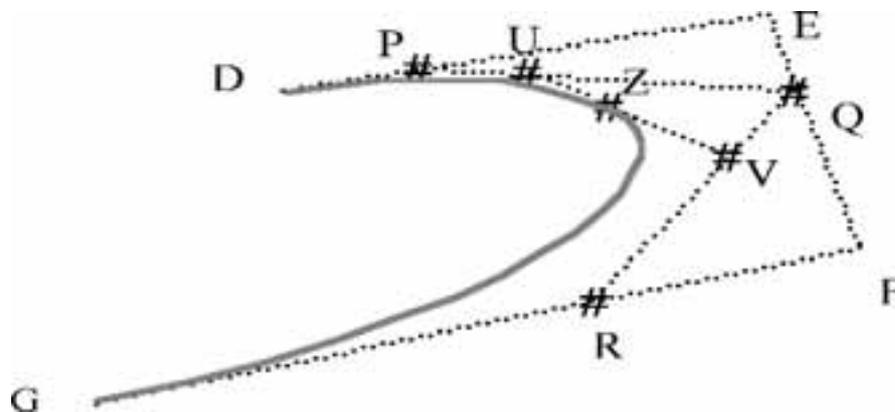
Figure 5. A Bézier curve as the locus of nested linear combinations.

After students have constructed linear combinations and Bézier curves geometrically, they can explore the effect of moving the control points D, E, F and G, and appreciate the geometry that is applied as the curves are edited in commercial software packages. With a greater understanding of the geometry, students can then go on to investigate the algebra of these curves using the symbolic capabilities of the calculator.

The algebra of Bezier curves

The availability of algebra systems on hand-held calculating devices enables the exploration of many concepts and processes. Many of these might previously have been regarded as complicated enough to prevent a thorough grasp of the material being accessible to many students. In addition to the algebraic capabilities, calculators that offer low-level programming, (similar to macros in computer packages) allow for student introduction to many concepts through use of pre-written exercises that allow for student discovery.

The TI-89 offers command scripts, which provide a mixture of explanatory notes and calculator commands that allow students to work through a derivation or proof, enabling them to focus initially on the concepts involved. Once the concepts have been absorbed, the opportunity for success in hand-worked processes is much increased. Below is a command script designed for students to discover the algebraic properties of Bézier curves.

```
:Enter control vectors
C:[1,0]→d                                    1
C:[4,1]→e                                    2
C:[0,1]→f                                    3
C:[3,2]→g                                    4
 :Define linear combination
C:(1-t)*x+t*y6lncmb(x,y,t)                   5
 :Find first level vectors p, q, and r
C:lncmb(d,e,t)→p                             6
C:lncmb(e,f,t)→q                             7
C:lncmb(f,g,t)→r                             8
 :Find 2nd level vectors
C:expand(lncmb(p,q,t))→u                     9
C:expand(lncmb(q,r,t))→v                     10
 :Find final, curve generating point, z
C:expand(lncmb(u,v,t))→z                     11
```

To access the components of z, open z in the matrix editor. To graph the curve, copy and paste the expressions from the matrix editor into the parametric Y= editor.

The text in the command script is entered at the calculator keyboard or can be downloaded from one calculator to another; alternatively it can be typed at a computer and then downloaded to the calculator from there. The script consists of explanatory notes (start with :) and commands. Those statements preceded by the letter C are calculator commands, and are executed as the script is worked through by the user. Each of these has been labelled numerically so that the output can be discussed below. The command script can be opened in the calculator's text editor and appears as shown in Figure 6.

Figure 6. The command script as it appears in the text editor.

The screen can be split to show both the command script and the home screen. This enables the user to see the immediate effect of executing the last command. This can be seen in Figures 7 to 9.

Figure 7. The split screen view after command 1.
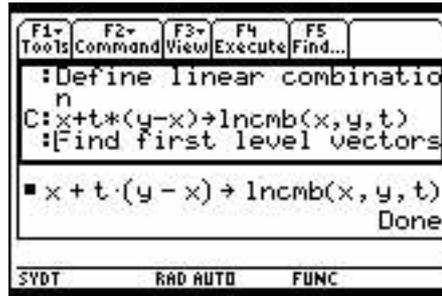
Figure 8. The split screen view after command 5.
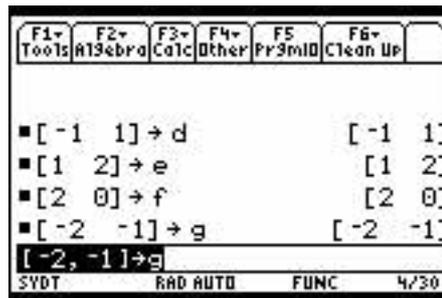
Figure 9. The full home screen after commands 1 to 4.

One aim of providing the command script is that it allows the student to work through the process of developing the algebraic expressions without getting lost on the way. This allows the student to grasp what is happening at each iterative stage, allowing them to attend later to the important questions of how and why it happens. By understanding what occurs, the student is more confident in dealing with the how and why issues. The development of the algebraic expressions at each stage allows the student to see the progression from linear to quadratic to cubic components for the position vectors in each iterative level. This is shown in Figures 10, 11 and 12 and 13.
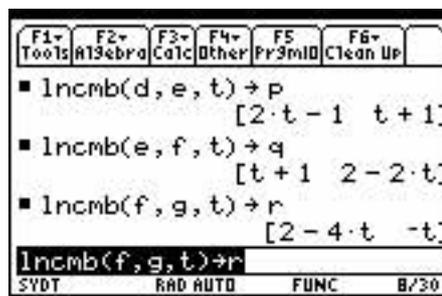
Figure 10. Linear expressions at level 1.
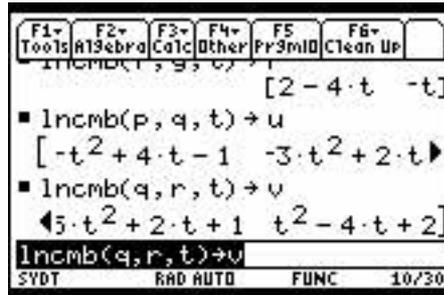
Figure 11. Quadratic components at level 2.
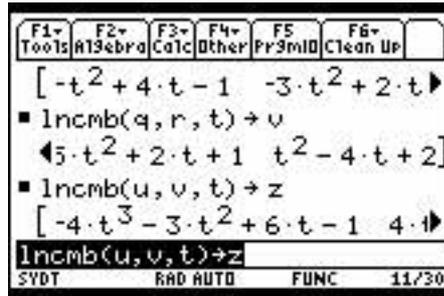
Figure 12. Cubic expression for the first component at the final stage.
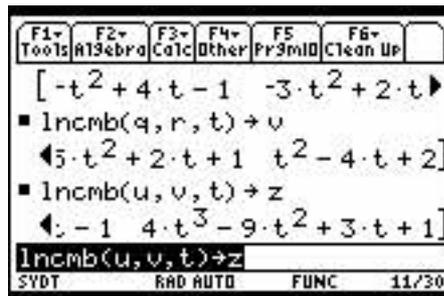
Figure 13. The previous figure scrolled right to reveal the second component.

As the final explanatory comment indicates, once the cubic components have been derived, they can be pasted into the graphing module of the calculator and used to produce the curve parametrically.
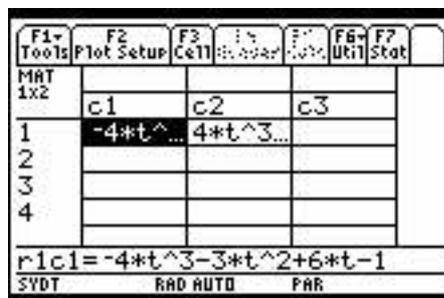
Figure 14. Vector z, as it appears in the matrix editor.
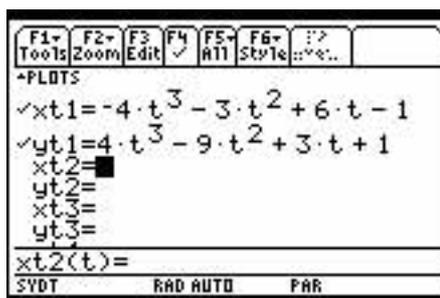
Figure 15. Components of z pasted to the parametric function editor.
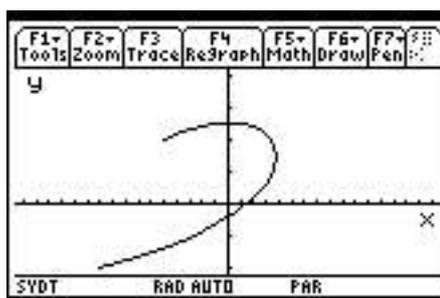
Figure 16. The curve produced algebraically, rather than geometrically.

Teaching mathematics from context

Often, traditional secondary mathematics courses have presented some mathematical techniques for students to 'master'. Students are then assessed on their facility with these techniques, before moving on to tackle the skills and routines in the next pre-ordained 'topic'. In recent years some attention has been paid to students undertaking mathematical investigations and projects in which the abstractly acquired skills are then applied. However, presenting the application after the mathematics means that for many students the initial mathematics has been studied in vacuo, and the motivation for mastery is missing.

A style of pedagogy that obviates the question 'When am I going to need this in real life?' can be a much more powerful way to motivate. Presenting the students with a context, such as Bézier curves in computer design, in which mathematics provides accessibility to a contemporary issue, and developing the mathematics as the need arises in the context under consideration, reveals mathematics as a powerful, descriptive and analytic response and, therefore, a very worthwhile and relevant field of human endeavour. The support that graphics calculator technology lends to such efforts makes it a very valuable educational commodity.

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Roberts, J. (1999). The Power of I.C.T. in the Delivery of Mathematical Concepts. In M. Bulmer, B. McCrae & K. Stacey (Eds) *Proceedings of the Mathematics 2000 Festival* (pp163–172). The Australian Association of Mathematics Teachers.

Summary and Reflections

Peter Gould

Department of Education and Training, New South Wales

In using technology to investigate Bézier curves, Jon Roberts demonstrated the value of using mathematically rich contexts as starting points. The investigation took in a wide range of mathematics including cubic equations, parametric curves, vectors, algebra and the process of recursion. The methods used in the investigation also highlighted some of the associated skills necessary in using a technology intensive approach. These demands include the requirements of visual literacy and the role of dynamic imagery in using programs such as *Cabri Geometry*. To borrow an analogy from John Mason, what is the effect of 'looking through screens' when developing concepts?

Jon pointed out how the technology itself can act as a lens — focussing students' attention and hiding the formative work of constructions that might otherwise divert student attention. Keeping in mind the desired endpoints of technology use in mathematics education, Jon emphasised the need to plan the introduction of technology use and to build the skills in the junior school. If you want students to be able to create a 'macro' command, you need to build up to this.

Our colleagues in the United States often stress the rule of three with respect to technology use in mathematics education: algebraic, graphical and tabular. This rule is founded on the belief that technology should support the conceptual links between alternate representations of the same situation — as an equation, a graph and a table of values. Jon's investigation of Bézier curves also brings to the fore the need to examine vectors within the twin domains of mathematics and technology use. Many of the latest applications of technology have returned to examine the benefits of vectors for storing information. If you double the size of a bit mapped image the memory requirements will generally increase fourfold. Using vector-based images removes this problem.

Some key questions for further investigation:

When is it better to use computers rather than graphics calculators?

What is the role of mathematics in developing visual literacy? Should we re-visit the foundation links of geometry and art in the development of perspective?

How does the need for authentic assessment sit with the use of technology in mathematics education?

Adding Value To Conceptual Learning

G. T. Springer

Hewlett-Packard Australian Calculator Operation, Victoria

The question of whether a child can travel faster on a bicycle than on foot is moot. It is not worth proving. The advantage of bicycles over shoes, however, depends on the terrain and even the age and physiological sophistication of the child... The question of whether a child can learn and do more mathematics with a computer (or other forms of electronic technology, including calculators...) versus traditional media, is moot, not worth proving... The real questions needing investigation concern the circumstances where each is appropriate.

This quote, from a 1989 article by James Kaput of the University of Massachusetts, summarizes the starting position of this paper. While there are still a few who continue to argue the point, the vast majority of those with practical experience in helping young people learn mathematics are convinced of the value that graphics calculators can add to the mathematics classroom. The focus of attention is identifying what the key advantages of graphics calculators are, and when it is appropriate to take advantage of these key benefits. The goal of this paper is to examine an activity illustrative of promising practices centered on graphics calculators, and to point out areas in which graphics calculators clearly add value to conceptual learning. There are two sets of opportunities presented by graphics calculators. The first set contains those activities that can be characterized as transfer of traditional pencil and paper activities to this new medium. The second set contains those activities that represent new opportunities afforded by graphics calculators. These latter activities either were simply not possible before, or were possible but significantly more difficult. It is from this second set of activities that our sample activity will be chosen.

One final note is in order, before we turn to the sample activity. The mathematics classroom in which we see value in using graphics calculators has certain characteristics, which include the following:

- students are actively engaged in constructing their mathematical knowledge;

- problem-solving is a means for constructing mathematical knowledge, as well as an end in itself.

These two characteristics simply point out that the learning of mathematics benefits from the richness that a valid context delivers. In our sample activity, we use an ordinary and familiar situation to construct mathematical concepts.

As a practical matter, mathematics is a science of pattern and order... As a

> *science of abstract objects, mathematics relies on logic rather than observation as its standard of truth, yet employs observation, simulation, and even experimentation as a means of discovering truth (Schoenfeld, 1992).*

The activity described in this section is a simple data collection activity. A Nerf ball is covered with foil and dropped from a height of approximately 2 meters. The ball drops onto a motion detector (see diagram below). The detector reads the height of the ball every 0.05 seconds. The time and height data are recorded and sent to the graphics calculator for presentation and interpretation. As the activity unfolds, note the use of both experimentation and simulation. Physical experimentation was possible before graphics calculators, but many experiments are now quite easy with data collection devices. On the other hand, simulation and experimentation with variety of representation, was simply not possible before.

Step 1: Collect the data

There are only three short steps in collecting the data:

1. Set the frequency and duration of the probe readings

2. Choose the probe and units desired

3. Start the data collector and drop the ball.

From setup to execution, the entire experiment takes less than five minutes! The ease with which experiments can be designed and executed opens many doors that simply were not open ten years ago. The migration to a more active and engaging mathematics classroom starts with encouraging the spirit of experimentation in our students.

Figure 1. Take 20 readings in 1 second.

Figure 2. Measure height in cm.

Figure 3. Collect height and time data.

Step 2: Presenting the data graphically

But experimentation is just the beginning. One of the greatest benefits of using graphics calculators is that they allow so much flexibility in the representation of mathematical objects. Just displaying the data graphically can be a source of creativity, as shown in the accompanying screen captures from a Hewlett-Packard graphics calculator. In the first graph, the data are represented as an instant replay of the ball dropping. The tracer can be used to replay the drop from start to finish, in slow motion or even reverse motion.

The student can move from graph as picture to graph as interpretive representation in a variety of ways. The second graph shows the positions of the ball as line segments. Clearly, this is no longer a 'picture' of the ball's motion in any direct sense. Rather, it is a representation that focuses the attention on certain characteristics of the motion. In this case, the increasing distance between segments jumps out at us. Finally, the traditional scatter plot of the (time, height) ordered pairs provides us with a clue that there is a simple, mathematical relation between these two data sets. Note that these three graphs move from a simulation, to mediated representations in 1 and 2 variables.

Figure 4. The data presented numerically.

Figure 5. Choose the columns to plot.

Figure 6. Set up the graph.
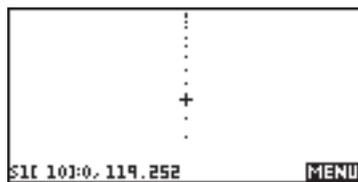
Figure 7. The data graphed as 'instant replay'.
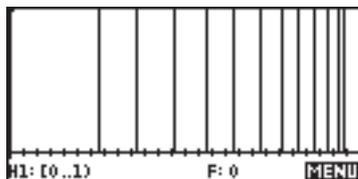
Figure 8. The data graphed as a set of markers...
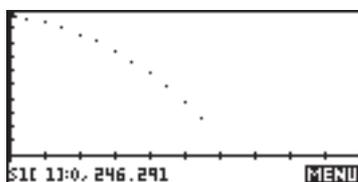
Figure 9. The traditional scatter plot.

Step 3: Building mathematical concepts

Of course, we've already started building mathematical concepts simply by varying the ways in which we presented the data. Moving to a symbolic definition of the relation between time and height for the ball-drop experiment offers more opportunities for graphics calculators to add value to the learning environment. Again, note the use of simulation and experimentation in the following discussion.

Suppose students have come to attach enough meaning to the second graph to understand that the (time, height) ordered pairs represent a quadratic relation. Suppose further that the students are comparing this situation to one they've learned about in their science classes. They would naturally want to know how close to an 'ideal' data set their data are.

Figures 10 and 11 show how students could experiment with variations on the formula $H = 246 - 490T^2$, in order to see how close their data are to ideal. C4 contains our actual height data. C3 contains the ideal data set $h = 246 - 490t^2$, C2 contains $h = 246 - 750t^2$, and C1 contains $h = 243 - 490t^2$. Students can speculate about the relative magnitudes of the changes needed in the 2 parameters in order to bring observation in line with an accepted model.
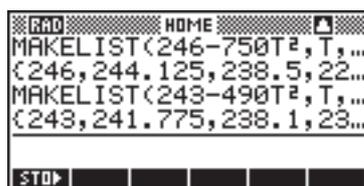
Figure 10. Experiment with 'ideal' data sets.

Figure 11. Compare and discover qualitative differences in the models.

The data we used in the previous two steps were actually collected (not synthesized) for this paper. The data sets were actually larger than those shown, containing initial values that were close to constant. These values were dropped as representing a brief interval in time during which the data was being collected before the ball was dropped. Thus, we are not sure if our first data point represents $t = 0$ or $t = 0.05$ seconds. In other words, students learn in a very real sense that there were two initial conditions: initial height and elapsed time until the first observation. The last screen shows C3 compared to our data in C4. This time, C3 represents $247.5 - 490t^2$, where t starts at 0.05.

Figure 12. Experiment with starting time.

Note that we have yet to use the calculator's built-in quadratic regression fit; rather, the students are allowed to fully explore variations in the model, noting which parameters are relatively more or less sensitive. Just realizing that time, height, and gravity can all play a part is no mean feat.

The foregoing activity and discussion have pointed out that graphics calculators can add value to a mathematics classroom in which active exploration and construction of mathematical knowledge is encouraged. Far from only providing computational advantages, graphics calculators can strengthen conceptual frameworks and encourage students to explore meaningful mathematics in a large variety of ways. If our goal is to instill the mathematical point of view in students, then the graphics calculator can be a valuable tool in every student's (and teacher's) toolkit.

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Summary and Reflections

Thelma Perso

Education Department of Western Australia

GT introduced his session by talking about research over the past decade into adding value to conceptual learning in mathematics as opposed to past (and present) approaches to pedagogy where student learn mathematical routines and procedures but often do not understand underlying concepts.

An historical perspective was given of the implementation of the graphics calculator with particular reference being made to the ad hoc way it occurred in the US through a number of events occurring simultaneously in 1989 which made their implementation or 'appearance' in an educational setting, timely. These were:

- Concern being shown by practitioners and researchers over Calculus instruction preparation, students having skills but little understanding;

- Invention of Graphics Calculators;

- The writings and disseminations on the theories of Constructivism;

- Development of geometry software; and

- Computer Algebra Systems.

These coinciding events caused teachers and curriculum developers throughout the US to sit up and ask themselves the question, 'What are we teaching in mathematics classrooms and how are we teaching these things?'

There was a realisation that there currently existed a routine, algorithmic approach to the teaching of mathematics, that many mathematics teachers were missing the point of teaching mathematics — that of APPRECIATION of the power of mathematics in problem solving. Included in this was a widespread recognition and acknowledgment that ALL children do not want or need to be mathematicians.

They were then forced to ask themselves the question, 'If mathematics teaching is about appreciation and developing a mathematical point of view, what are the 'tools' of the mathematics teacher and student?'. The conclusion was: 'It is not SHOULD we be using a calculator but WHEN and WHERE should we be using it?'.

A model for mathematics instruction was sought. The NCTM, in 1989, put forward a view to actively involve students in constructing their own knowledge and to use problem solving as a means as well as a goal. They acknowledged that learning environments benefit from a rich environment that a valid context provides.

It was recognised that graphics calculators can create new opportunities that strengthen or enrich mathematics learning. The opportunities include:

- the everyday classroom experiences such as collecting data from the physical world

with peripheral devices such as a motion detector or a probe;

- opportunities for making connections between number, concepts and understandings — moving away from the controlled environment and expectations for the learning to a setting in which unexpected 'Aha!' understandings can and do occur;

- opportunities for transfer from school mathematics to the real world, hence enriching appreciation for mathematics;

- opportunities for student individual investigation and making connections across systems; students observing and making mathematical discoveries on their own. For example, the transformational approach to graphic functions allows exploration of the relationship between equations and the curves they generate.

Minor Presentations

Hand-held Technology in the Future: Probabilities and Possibilities

Stephen Arnold

St Mary Star of the Sea College, New South Wales

The rapidly-evolving nature of technology means that our knowledge of the past illuminates very little of our future. The development of hand-held technology for mathematics teaching and learning over recent years nonetheless offers some clear insights into likely directions in the future — at least in the short term. The likely nature of this technology for educators forms the focus for this presentation.

Attempting to predict the future is a thankless task: at best, it is likely to be futile; at worst, it may even be dangerous, limiting our vision of what could be. Our predictions, of course, are based upon what we know of the present: as Seymour Papert (1980) aptly put it, 'The first instinct of educators is to couple the new technology to their old methods of instruction' (p. 230). Our perceptions of the nature and possibilities of technology are necessarily constrained by our own experience and ability to imagine alternative realities. Papert relates the story of the introduction of movie cameras and motion pictures: initially, the cameras were set up in front of the stage as the players performed in their traditional ways. It was a long time before the art form transcended these limitations to produce something new and unique, where camera, performer and set become inextricably intertwined. The same may be expected of technologies for teaching and learning. At present, with less than two decades from which to learn, we are still very much at that early stage: carrying out our teaching and learning processes in very much the same way as we did before the development of computers and graphic calculators. The technology remains largely peripheral to the learning experience instead of being at the heart of it.

With these warnings in mind, we may step further into our predictive mode as we try and envisage the nature of the technology as it will be in, say, five years. The short-term nature of our task ensures that we may not expect enormous changes. The nature and purpose of hand-held tools for mathematics learning may be expected to remain essentially the same as we currently experience them. It is even safe to assume that those features which at present mark out the 'cutting edge' (symbolic algebraic manipulation and dynamic geometry, real-world interfacing through data logging, for example) of the technology will provide us with a framework for predicting the likely norm in five years. But even here we must tread carefully.

Some fifteen years ago, the first graphic calculators were released: essentially, scientific calculators which could draw graphs. By 1987, I was using in my classroom calculators which could not only graph functions, but which had quite sophisticated algebraic manipulation, calculus and equation-solving facilities. These Hewlett-Packard HP-28 series devices seemed to promise much: granted, the symbolic manipulation facilities were quite

difficult to use, but it seemed that it would only be a short time before such features would become the norm. Certainly, within a decade, the wonderful TI-92 from Texas Instruments offered a device which more than fulfilled the promise of these early algebra machines, and yet, at the start of the new millennium, we remain firmly wedded to second generation tools (characterised by capabilities for graphing, tables of values, statistics, numerical calculus and equation solving, and programming) and still some way off accepting the third generation tools now available.

While it is tempting to predict that computer algebra and dynamic geometry facilities will be the norm in five years, our experience of the past fifteen years suggests otherwise. We are held back in this regard, not by the technology, but by more political and pragmatic concerns, especially those related to external assessment issues.

So what can we expect? What we are likely to see within the next five years builds upon another cutting edge development known now as 'Flash' technology. This allows additional 'apps' or applications to be downloaded from the Internet or disk and uploaded to significantly increase the functionality of the calculator. These Apps are far more than the simple calculator programs which teachers and students have been writing for some years. They are full-featured tools which fundamentally change the capabilities of the device. Consider, for example, the Texas Instrument TI-89. Released a couple of years ago as essentially a TI-92Plus without the geometry and the QWERTY keyboard, it may now be enhanced by adding the full *Cabri Geometry* package or even the *Geometer's SketchPad* calculator version. Also available are Advanced Algebra packages, data-logging apps — even ROM updates may be downloaded and installed in this way.

Prediction 1

> Within five years, students and their teachers will buy a 'shell calculator' (probably offering most of the 2nd generation features presently available) and then be able to extend its capabilities in a modular way, adding algebra, geometry, data-logging or other facilities as required. The tool will become enormously flexible and, as a result, increasingly affordable.

Prediction 2

> It seems likely, too, that the interfacing of personal technological tools with the real-world will only increase, becoming simpler and more accessible as it expands our vision of what and how we teach in the name of 'mathematics'.

The final prediction concerns the wedding of the two great technologies for learning of the present: the Internet and hand-held devices. Infrared networking is now sufficiently developed that we can expect it to be a standard inclusion on all hand-held devices.

Prediction 3

> Within five years, all such calculators will have infrared network capabilities (as do HP calculators at present) which will enable them to be used as communication devices, certainly supporting electronic mail and the ability to upload and download data and software updates and extensions. Again this will raise enormous issues regarding use for assessment and the limitations which we feel the need to place upon such use.

Clearly, the implications for teaching and learning are enormous. As always, our effective use of such tools will be limited, not by the technology itself, but by those who would use it. Let us anticipate the future and be prepared to welcome the possibilities!

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Calculating the Future:
Threats and Opportunities

Peter Galbraith

The University of Queensland

The use of the plural (as in the title) is deliberate by those who work in the Futures field, since their conviction is that the future is there to be influenced, changed, or even created, not simply accepted or imposed. The fundamental principle is that of 'choice': that alternative futures being possible, the eventual outcome can be influenced by human input.

Thus when we think of future roles for educational technology (such as graphical calculators) we must inevitably consider the part we may play in their emergence, sustainability, or demise. So this contribution looks at possible futures for graphical calculators, not from the viewpoint of the mathematics they can accomplish, but from that of a potentially transforming innovation subject to positive and negative influences.

Sustainable growth

It is useful to consider the growth pattern that emerges when a rapidly growing quantity is impacted by a finite environment. This is the familiar sigmoidal growth, that describes the growth of an organism, the spread of an infectious disease, the sale of a popular new product, the spread of an innovation and so on.

The fundamental growth pattern and the underlying growth structure are shown in Figures 1 and 2 below. Units on axes are arbitrary.
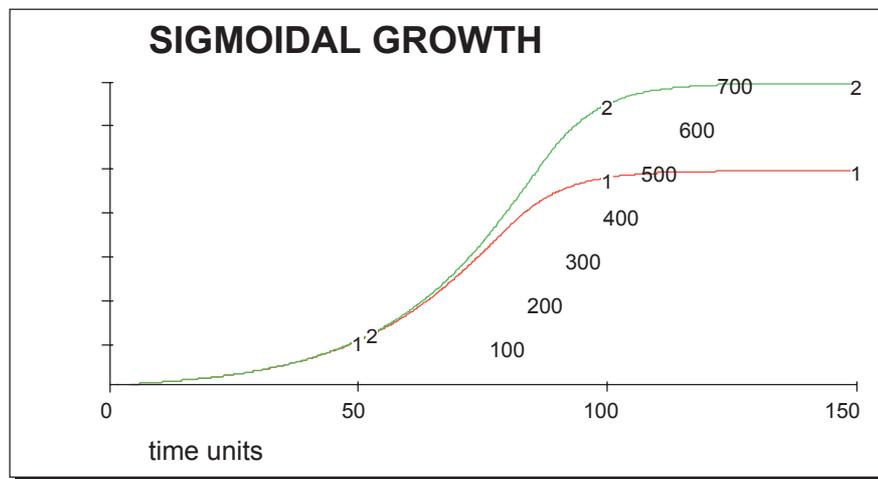
Figure 1. Growth curves.
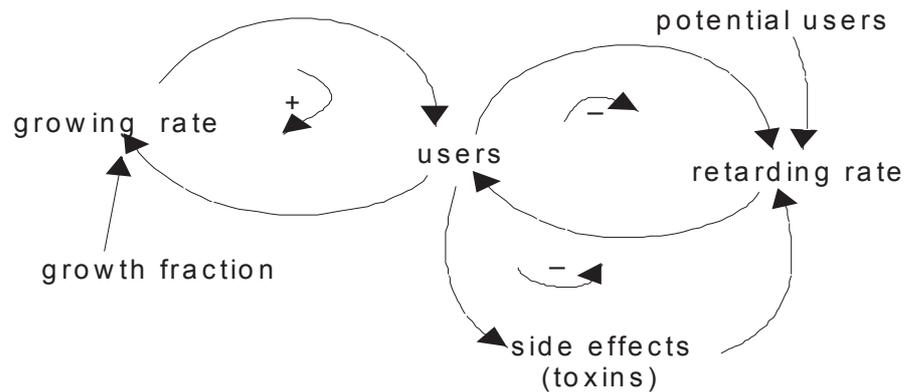
Figure 2. Structural diagram.

The compound interest loop on the left describes the initial phase before the growth limit becomes a factor, when numbers increase in proportion to the population. The top retarding loop depicts the process that curtails the growing rate, as the difference between present population and potential population decreases. If no other factors are at work the population will approach the carrying capacity as shown in graph 2.

However, other factors are very often at work: a growing organism produces toxins in proportion to its numbers so that its influence (as in the lower retarding loop) can cause growth to cease before carrying capacity is reached, as in graph 1. Of course these graphs are ideal types, and actual growth patterns are lumpy, but the behaviour mode is robust.

Two matters of significance can be noted. Firstly growth in a finite environment cannot maintain buoyant levels indefinitely: at its best, it must asymptote to the carrying capacity defined by the potential population. Secondly side-products (toxins) may be produced which inhibit the growth so that this potential is not realised.

Applied to social contexts there are other influences at work. Below the point of inflexion optimism reigns as the rate of growth continues to increase. The world is buzzing as new members join and the 'movement' grows. When the gradient passes its maximum, as it must in a finite world, there is a sense of loss of momentum. A possible social outcome is the feeling that the innovation has had its day, when in fact it is following a natural growth pattern and much remains to be done. Simultaneously social side effects may be produced with toxic effects on growth; just as inhibiting as the waste products that impede the growth of biological populations. In fact around the inflexion point is where a new generation of development is likely to begin in response to the perceived slowdown: two creative processes with different needs and in different phases of development exist together and each needs separate supportive encouragement.

Let us apply this thinking to the growth of an imaginative technology, which for present purposes we shall take to be the use of graphical calculators. Let us assume that the enthusiasm engendered among pioneering practitioners, together with support from official sources, generates increasing numbers of users such as is presently occurring.

It is fair to say that with respect to the growth model we are still in the very early stages of increasing growth, and that we can expect this growth to accelerate given present moves within the profession and among accrediting authorities.

How will future usage patterns emerge? As a coarse approximation we could consider the potential capacity as the number of classrooms in which mathematics is taught. However various other bounds can impose lower limits on actual growth.

A most obvious limit is the availability of facilities and hardware, and clearly a lack here will curtail growth. But suppose there are no such physical limits, and schools are resourced so that every classroom is provided with the necessary equipment. Now growth is no longer limited by hardware, and a new potential limit emerges as the overall skill level of the practitioner population. Suppose now that training programs can provide expertise so that in theory this limit is lifted also. Inhibiting forces are almost certain to remain and they are not the consequence of either physical or skill limits. The rapid initial growth takes place among enthusiasts, who wholeheartedly accept the new opportunities and generate an enthusiasm, which feeds the growth further. However, resistance will be encountered from continuing non-converts. This resistance may be philosophical, based on sincere concerns that important goals of mathematics learning are being compromised; or psychological as a reaction to the excessive enthusiasm of peers, and perceptions that valued forms of learning are being extinguished or marginalised. This latter might be recognised as an analogy of toxin production, that as a product of existing growth, acts to inhibit further growth. The overall point is that growth-limiting conditions can shift between physical, skill, philosophical, and psychological influences. Each needs to be addressed, and the necessary methods are different.

So following a belief that the future is there to be created what implications for action emerge from the above? There is a guiding principle espoused by those who work in systems that face such issues. This principle is not to fight against the limits, but to seek ways to relax or remove them — the antithesis of 'crash through or crash'. What the profession can do is continue to give invigorating leadership that demonstrates the advances that can be achieved through a combination of hardware and skill. In particular, principles of sound mathematics and pedagogy need to be articulated, promoted, and consolidated beyond the level of individual 'show and tell'. This is necessary not only for addressing limits due to equipment and skill deficiency, but also those with philosophical or psychological bases. Debates on the nature and quality of mathematics need to be engaged as an essential part of the whole, and the assembly of weight of evidence and soundly based mathematical rationale is an indispensable component. If such cannot be convincingly employed then opposing arguments must have a point! Arguments that use intellectual blackmail such as 'you are out of date' deserve to fail.

Growth and collapse

While limits can prevent an innovation from reaching its potential, a greater long-term risk is that growth will peak and collapse will follow. The loop structure would be distracting to include, but the behaviour is illustrated in Figure 3.

A familiar example is found in the history of 'ghost' towns where populations grew rapidly, attracted by stories of 'gold in the streets'. Eventually it became clear that expectations could not be met by production, the cost of extraction became more than it was worth, mines closed and people left. Often hopefuls continued to arrive even as the viability of the enterprise was coming into question. (The growth of the gold town Walhalla in Victoria

resulted in a railway link being opened in 1910 — the town collapsed in 1913). The difference from the previous case is that in the case of sigmoidal growth a sustainable resource supported a stable carrying capacity, with new users replacing those who left, to maintain an overall steady state. There the challenge was to remove limits restricting the growth of a quality resource; here it is to avoid the consequences of failing to meet the expectations of those who have joined the cause.

GROWTH AND COLLAPSE

time units
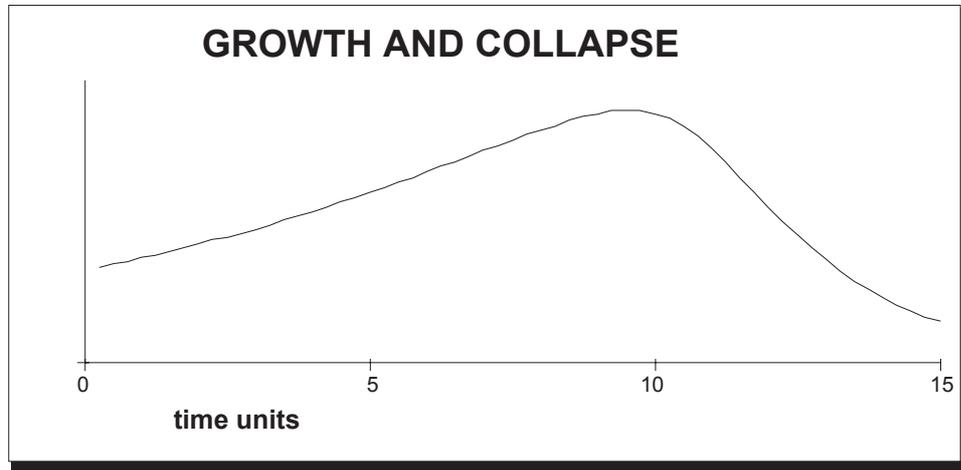
Figure 3. Growth-collapse curve

Can a technology like graphical calculators meet the future challenge of retaining its public? It will mean being able to fulfil promises as expected by users; the educational equivalent of 'gold in the streets' promises. Enthusiasm and 'samples' will not be enough. The product must be able to make the effort worthwhile long-term; that is retain and enhance its value for effort ratio in comparison with alternative teaching technologies and approaches. This will require sustained development of readily accessible and updated teaching resources, continuing provision of professional support, and sustainable high quality achievement. Such a challenge will be on going and systematic, for we can reflect on previous educational victims of growth and collapse. As a movement we might identify 'New Mathematics'; as a technology 'Programmed Learning'. (Note there is no value judgment being made as to the worth of these enterprises).

Destabilising enthusiasm

One of the most effective means of dampening the spread of a new idea or technology is the diversion of those who are key players in the initial growth phase. This tactic has several faces and many motivations. If I do not want to follow in your innovative footsteps, one tactic is to deprecate your interests by pointing to some new embryonic laboratory technology that has made the news. I then point out that this will render your activity and interest obsolete and thus avoid any need to follow the direction you have set. In addition to enjoying some ego massage, I will probably have cast doubts whether all the effort you are applying is worth continuing! Table 1 lists a variety of technologies.

TECHNOLOGY	Invented	Years to innovate
Chemical fertilizers	1840	45
Gasoline motor	1860	26
Power steering	1900	30
Radar	1887	47
Television	1907	29
Silicon chip	1958	approx. 30

Table 1: Selected Technologies

Invention refers to the first laboratory production of the basic technology: innovation to its widespread commercial impact following development. (Invention of the integrated circuit has been taken as occurring within a Texas Instruments laboratory, rather than the Roswell conspiracy theory that it was invented by aliens sometime before 1947). The point here is that development time is always substantial, of the order of a current working lifetime. This is a commercial variable, not a technological one. So the kind of argument outlined above, if presented as a distraction from a productive line of activity, should be carefully evaluated and in most cases ignored. It is important that practitioners feel their environment is stable enough to support the sustained activity that a carefully selected technology warrants, and that ultimate success requires. As one innovative team expressed it, having experienced efforts by companies to introduce extravagantly promoted programs.

> *Electronic classroom builders may create showcase environments that impress visitors or donors, but their pedagogical vision is often lacking and implementation flawed* (Shneiderman et al., p. 25).

Perhaps these can be identified as cries by commercial wolves of 'gold in the streets'! Of course continual change and improvement within a carefully selected technological genre is to be expected and welcomed.

Emergent properties

In looking to the future it is easy to think about technology such as the graphical calculator exclusively in terms of mathematics. In fact we are beginning to see evidence that the interaction between human agents (students and teachers) and technologies is creating new pedagogical patterns and opportunities. These involve contributions from technology that were not obviously part of their original design purpose. Recently I saw a mathematics lesson in which the students were moving between the following: a Web page on numerical methods of integration, their graphical calculators, pen and paper observations and checks, and collaborative talk. New patterns of classroom activity and relationships were developing. The exploration and expansion of such possibilities adds a whole new dimension to the potential value of technologies in teaching, one that extends well beyond that of a purely mathematical servant. These emergent properties also need to be documented and evaluated as factors in taking mathematics teaching and learning to the next level, and in supporting the introduction and sustaining of quality technologies.

Into the future

Recently I read an article in which the author used an interesting metaphor. A question was posed as to how the future of the transport industry might have been predicted in 1910. Not much future could be seen for cars it was suggested, because the roads were all ruts and puddles, and no extensive network of highways existed. Aeroplanes were embryonic and unreliable, and few would put their lives and businesses at risk by depending on them for transport. Well! Well!

What we see of course is the limited view that results when the potential of new technologies is assessed in terms of existing infrastructure. And there is no less a propensity to do the same in education, for several writers such as Ramsden (1997) have noted the impact of inherited traditions on the use of technology; by referring to an instinct for teachers to begin by looking for electronic ways of doing familiar jobs previously done by textbooks and lectures. This position is supported (Thorpe, 1997), who, in examining teaching behaviours and attitudes towards technology, found that technology was being used essentially to enhance preferred existing teaching methods.

So let us not make the same mistake here. Rather than judging the potential of a technology such as graphical calculators against the backdrop of past and current practices, and present structures, the opportunity exists for changing some of these practices and structures in order to achieve the potential offered by a critically responsible use of the technology. Some are already achieving this; the extent that it can become a major transforming agent for the future will depend on how successfully its accomplishments and supporters can sustain momentum in a world of continuing educational challenge and competition.

The 'future' in a real sense is in the hands of the profession. Making it a productive one will involve abilities to read and interpret 'growth properties' in different phases of development, and to address potential impediments involving both technological and human agencies. It will also involve servicing the expectations of those attracted on the basis of 'promise', and whose continued involvement is based on perceptions of benefits received per effort expended. This can only be helped by the systematic gathering of evidence, and the continually updated provision of quality materials and methods, based on consistent rationales featuring both mathematical and pedagogical integrity.

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How Can Graphing Calculators Stimulate Collaborative Inquiry?

Merrilyn Goos

The University of Queensland

Overview

It may seem surprising to include the topic of collaborative learning in a conference on graphing calculators, as one of the greatest advantages of these devices is that they provide students with individual access to technology. However, evidence emerging from classroom research shows that graphing calculators can be a powerful stimulus for collaborative work (Galbraith, Renshaw, Goos & Geiger, 1999; Goos & Geiger, 1999).

This paper focuses on three key issues:

- emergent properties of technology

- the nature of collaborative inquiry

- challenges for teachers.

Emergent properties

It seems natural for teachers to use new technology in ways that are consistent with preferred teaching methods. However, teaching with graphing calculators need not — and perhaps should not — be a matter of simply grafting the technology onto existing pedagogical practices. For example, likening the graphing calculator to a portable computer, and the overhead projector panel to an electronic blackboard, obscures important qualitative differences between old and new technologies and may limit the scope of what teachers and students are able to achieve in the classroom.

In contrast, research that seeks out emergent (i.e unplanned, unexpected) uses of technology reveals that the calculator and OHP panel are not passive or neutral objects, since these technologies are actively re-organising human interactions and interactions between humans and the technology itself (Galbraith, Goos, Renshaw & Geiger, forthcoming).

Collaborative inquiry

Collaborative inquiry may involve social interaction in both private (e.g. small groups of students) and public (i.e. whole class) modes. In the former context, the role of the graphing calculator in stimulating mathematical discussion could be described by the

following metaphors:

- Message stick — students pass their calculators back and forth amongst the group, or hold up two calculators side by side in order to display and compare their working.

- Notepad — students can easily amend calculator working to correct their own and each other's errors or investigate 'what if' questions.

- Partner — students display intense emotional involvement with the calculator, often talking to it as though it is a partner that responds to their commands.

A natural extension of this private collaborative discussion occurs when students are invited to present their calculator work to the whole class via the overhead projector panel. When control of the discussion is handed to students the OHP panel is no longer simply a presentation tool — instead it becomes a discourse tool that mediates interaction between students at the whole class level. For example, students may use the OHP panel to demonstrate, compare, and improve programs they have written, with questions and comments from the class being directed to the student presenter rather than the teacher. This is a form of class wide collaborative inquiry that is facilitated by the public display and interrogation of mathematical ideas.

Challenges for teachers

Introducing graphing calculator technology into mathematics education changes classroom interaction patterns and the ways that knowledge is produced. Implicit in these changes are three challenges for teachers.

- *Technological challenges*

  The most obvious challenge involves becoming familiar with the capacities of the graphing calculator and peripheral devices. While this will remain a significant professional development focus, other issues also deserve our attention.

- *Mathematical challenges*

  Once we place graphing calculators in the hands of students we give them the power and freedom to explore mathematical territory that may be unfamiliar to the teacher. Is this challenge to teachers' mathematical expertise and authority something to be avoided or embraced?

- *Pedagogical challenges*

  This final challenge may be the most significant of all — how does one orchestrate collaborative inquiry in a technology rich classroom? And if graphing calculators are welcomed as a liberating opportunity for students, then does this freedom extend to use of the overhead projector panel? Such questions go to the heart of our pedagogical beliefs, and our beliefs about the nature of mathematics.

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The Student of the Future — the Sky's the Limit ... But What About the Glass Ceiling?

Anne Hastings

Sydney Church of England Girls Grammar School, New South Wales

It was not gender differences that I had in mind when I wrote the title of this short presentation. I do not believe that the limitations on learning with technology are currently, or will in the future, be gender dependent. In my experience, girls are every bit as confident as boys in their use of technology as a tool in learning. All they need is the opportunity — that is, access to technology and teachers who will facilitate their learning through the technology, helping the students see the possibilities and allowing them to realise their potential. My reference to the glass ceiling relates to the great inequities in access and opportunity that are currently occurring.

Let us ponder for a moment on the dark past of technology-free learning. When I started teaching a little over twenty years ago, the Canon Canola (with 7 memories) was the current technology. This was soon replaced by a computer that stored information on a cassette tape (seen as an amazing breakthrough at the time). Even ten years ago email and the Internet did not exist for most of us. Five years ago we were just beginning to use these facilities but had real trouble using technology in the mathematics classroom. There just were not enough computers to go around and the graphics calculators were still seen to be too expensive. The changes that we have witnessed in the past twenty years have been enormous.

In beginning to think about the student of the future I turned to the K–6 section of my school to analyse these students' use of technology. These students (girls) are using computers as tools for communicating, creating, accessing information, organising, tutoring and enhancing learning. Examples of their use of technology are:

- Kindergarten girls are creating a multimedia presentation using the computer to draw, write and explain the difference between 'thick' and 'thin'.
- Year 3 girls are creating a hyperstack to record their research on important scientific achievements.
- Year 5 girls are developing a database to store and present information about species of plants, including pictures of each plant that have been downloaded from the internet.
- All students use the internet as a matter of course for research.
- As soon as they can write they use email for communication, both within and outside the school.
- They use word processing skills for presenting their projects from a young age.

There are at least six computers in each classroom and they access the computer laboratories in the senior school on a regular basis. These girls are technologically literate and they are my 'students of the future'. It would be irresponsible of me to ignore their previous background and neglect to continue their use of technology as a tool to accompany and enhance their learning.

In their future, they will be using technology for work, home, leisure, learning, communication etc. on a daily basis. Will they ever need to go into a bank with all the facilities of online banking? Will shopping become a thing of the past with all the e-commerce possibilities opening up? Emails can currently be done from the beach with the latest pocket sized technology; voice-recognition technology is much more common; video telephones are being used by 'ordinary people'; and everyone over 12 seems to have their own mobile phone! What will we see in the next 5–10 years?

Their use of technology benefits students' learning in many tangible ways. Individual needs are catered for quite readily, students are learning at their own pace and get immediate feedback.

Geographical boundaries are broken down, since it does not matter where the student is, they can still access files and work that other students are doing. For distance education, the use of online technology can remove the disadvantages of remoteness, with ready access to guidance and resources that were not previously available.

Students learn mastery of technological tools by trial and error and thus are more willing to take risks. They have more control over their own learning and so are empowered by their use of the technology. These aspects have considerable implication for teachers since learning by 'trial and error', taking risks and handing over some of the control to the students can be outside the safe square of teachers' experience.

So what skills do students really need for their future? To be able to cope with the demands of their lives in the rapidly changing society, they need to be able to think critically, to be selective and independent, adaptable to new situations, good problem solvers, clear communicators and to have a strong foundation of skills. Teachers have a very challenging and important role here to assist the development of all these aspects of our students. In relation to technology, teachers need to work towards making students aware of the opportunities and possibilities, act as a role model through their own use of technology, ensure student acquisition of appropriate skills, support and guide student learning, but at the same time stand back to allow student autonomy, creativity and achievement of potential.

The real glass ceiling of limitations for many students comes from a lack of access to relevant and useful technology or less than adequate opportunities to enhance their learning with technology. Not only does this apply to the physical resources of computers and graphics calculators, but also to human resources. Many teachers are still in the dark ages of technology-free learning 10–20 years ago for a variety of reasons such as a lack of physical resources and professional development. Failure to overcome their traditional attitudes towards learning and to take risks in an area they may not have fully mastered, as well as insufficient time in which to really understand innovations and for lesson preparation, lead to a less than ideal approach to the use of technology.

Until these challenges are addressed, the student of the future (e.g. our kindergarten girls) will be up against the glass ceiling by the time they are in secondary school. Those who do not have the access or opportunities may find that it is lower and made of steel!

Some Words of Caution about Graphics Calculators

Bill Pender

Sydney Grammar School, NSW

Note: This is an edited version of an address delivered to the AAMT Conference on graphics calculators held in Sydney on 30–31 March.

I am feeling a bit out of place here, perhaps like a monarchist at an ARM meeting, or a republican at an ACM meeting — take your pick. I should say first that I have appreciated many of the excellent teachers who have spoken here, who have been full of ideas and enthusiasm for Mathematics teaching. I believe that graphics calculators have been important in convincing many pupils, and many teachers too, that a function has a graph, and that the graph is an important tool in the study of that function. Of all that has been said, the remark which most impresses me is that pupils with graphics calculators can experiment with them independently of the teacher — I know very well that playing is indeed an important part of learning.

But since I have been invited to attend, I think it is appropriate that people hear at least some aspects of a far more cautious view. Attitudes like mine are shared by a great many experienced and successful mathematics teachers in NSW and by leading research mathematicians at our universities. They are not ill-considered views.

Freedom, good teaching, and authoritarianism

My first point concerns compulsion. I believe that capable teachers who know their subject should have freedom to teach as they see appropriate, with as few constraints as possible. It is good to hear the commitment expressed here for new ways of teaching with graphics calculators. On the other hand, many NSW teachers I know have become committed to using computers in their teaching, and are not at all interested in graphics calculators, regarding them perhaps as ugly and inflexible, or as an outdated machine having little to do with real modern technology — remember that they are not designed or used for any task other than teaching mathematics. Many others, like myself, have made an informed choice not to use graphics calculators or computers in their teaching very much at all, believing that most of the time they can convey the essential methods and structures of mathematics better without them.

In my own case, I make constant use of computers, particularly for typesetting, which can require some tricky programming. I believe that some use of computers may assist me a little in teaching some topics, and I will probably move further in that direction. But I do

not want my Board of Studies telling me how to run my classes. Teachers should be free to experiment, to choose methods which suit the ability, motivation and character of their classes, and to follow their own interests — this I believe is the way that properly educated teachers teach best.

I am concerned at the authoritarianism of some of the policies proposed at this conference. 'Teachers should be using graphics calculators, and they won't use them unless they are compulsory.' 'The government won't provide resources for graphics calculators unless they are compulsory for everyone.' 'I want to teach with every pupil using a graphics calculator all the time, and I can't do this unless they are allowed in the examination, so everyone should have them in the examination.' I and many others have little sympathy for these views. It is worth noting that 50% of the assessment in the NSW HSC is internal, and as far as I know, there is no reason why graphics calculators cannot be used for every internal assessment task. Surely this gives people sufficient scope and choice.

The scope of these remarks

My remarks here are restricted to those who achieve in the top 30–40%. Less academically able pupils who are not capable of going forward in mathematics need applications which are useful to them, and I am equally cautious about graphics calculators with this group. But the arguments are different — for example, there is a strong argument that they have a more urgent need for mathematics to be integrated with proper mainstream technology such as spreadsheets — and I will not pursue those issues here.

The purpose of a mathematics syllabus

For our more able pupils, the purpose of Years 11 and 12 is intellectual foundation for further study. They need experience in literacy criticism, they should ideally take a language, they need some training in the sciences, they should learn music if possible, and they should understand mathematics as far as they are capable, and so forth, as far as their ability permits. The overall purpose of their HSC study should be to gain some acquaintance with the characteristic methods and structures of a variety of disciplines.

What then is the essential nature of mathematics? My understanding is that it rests on the complete unity of two complementary aspects — structure, and the understanding of why that structure arises. In their highest form, these two aspects are called 'theorem' and 'proof', but we know them more commonly as 'the answer' and 'the solution', or as 'the result' and 'the reasons for it'. This understanding of mathematics is already the basis of Euclid's famous textbook, and it remains the characteristic of the subject to the present day. To say it more simply, getting an answer without knowing why may be interesting and useful, but it is not mathematics.

The really valuable intellectual experience to be gained from mathematics then is the experience of complete mastery and understanding of a connected structure of thought. Life is not like that, but mathematics aspires to be. With the current NSW mathematics courses, I can convey this, in a manner appropriate to the class, from our ungraded Year 7 classes to the top 4 Unit class in Year 12, and this is the reason I have supported our syllabuses so strongly in recent years.

The construction of a mathematics syllabus

How does one construct a syllabus? Mathematics is an unbounded system, so one needs to take a small and closely defined part, and create there a microcosm of mathematics as a kind of nursery where the methods and structures of the discipline can be learnt.

The obvious choice of subject for Years 11 and 12 is elementary calculus. My background is group theory and linear algebra, but both are far too hard — in all my teaching, I have really only been able to teach anything significant about group theory to one pupil — as are things like combinatorics or abstract algebra or projective geometry. But calculus, besides being extremely practical, provides an even, graded climb. Algorithms and interpretation are constantly intermingled, and almost everything can be realised geometrically, so that there is a good balance between analytic and wholistic thought. Almost everything can be unified under the heading of degree 2 phenomena through their relationships with the geometry of the various conics, and the basic objects of the course are

$$\frac{dy}{dx} \qquad \int f(x)\,dx \qquad x^2 \qquad \frac{1}{x} \qquad \sin x \qquad e^x \qquad (x+y)^n$$

I would invite people to look at the past HSC papers in NSW, particularly those from 1990. They can see there the impressive way that the unity of the subject is made clear and its methods and structures faithfully represented — how geometric and algebraic understanding are integrated, how the relationships between algorithms and understanding are constantly explored, and how the most inventive movements are required from the algebra to the graph and from the graph to the algebra. These papers are, I believe, the best that is achievable in school mathematics courses, and they depend on a continuing tradition of strong and capable academic mathematicians working in close cooperation and mutual respect with effective classroom teachers.

Graphics calculators as a cheating device

Here then is the problem with graphics calculators — and I realise that these are tough words. Given any finite piece of mathematics, it is possible to build a machine to model it. For example, HSC mathematics is a very small subset of the well-known computer programs *Mathematica* and *Maple*. Such a machine may, or may not, help people to learn the subject. But if you introduce it into the final written examinations, you are short-circuiting the very process you have undertaken in constructing the syllabus in the first place. You no longer have the microcosm of mathematics, the nursery where pupils can learn the discipline — the machine is functioning now not as a learning tool, but as a cheating device, undercutting what you are trying to achieve.

Curve sketching and the knowledge of functions

The 1998 Western Australian final examinations were the first in that State to allow graphics calculators. A structured question was asked about the function

$$y = \frac{1}{1 + e^{-x}}$$

Incidentally, I have taken this function from an article in the recent AAMT journal discussing the effect of allowing graphics calculators in this examination (Forster and Mueller, 1999). I would like to recommend this detailed and most interesting article. For me it is a strong argument against graphics calculators in examinations — the confusion of methods used by the pupils is extraordinary.

In any examination question on this function, I would want to begin with some analytic questions:

- What is the domain?

- What symmetry does the function have? (very difficult in this case)

- What are the zeros, and what is the sign?

- What are the asymptotes?

This could be followed, if one wished, by questions about the first and second derivatives. Then I would want all this analysis to be gathered together in the sketch of the graph, which gives a holistic view of the function and its properties.

But when graphics calculators are permitted, the graph is obtained first without any thought, and most questions about analysing the function are obvious from the graph, apart from irritating and irrelevant problems related to the size of the pixels and whether the pupils can use the various buttons effectively. No doubt one can ask some interesting bits and pieces, but the systematic structure is gone — as a test of knowledge of mathematics, the question falls apart.

More generally, I have two expectations of pupils' knowledge as regards the functions contained in our courses.

- They should know how the basic functions like x^2, $\dfrac{1}{x}$, e^x and $\sin x$ behave, and they should know what their graphs look like.

- They should know how to reason from the properties of these functions to the properties and the graphs of compound functions built up from them.

The whole HSC syllabus in NSW is based on this approach to functions, and it would become completely impossible were graphics calculators permitted in examinations.

Graphics calculators and scientific calculators

Here is a clearer way of saying it. Why shoot ourselves in the foot by eliminating our most basic examination question,

Sketch the graph of the function whose equation is $f(x) = \ldots$

You cannot examine an introductory course on 'functions and graphs by using a graph-drawing machine' — the machine can only be used when knowing the graphs is no longer an issue and the attention is on something more sophisticated.

The same principle should have been applied to the scientific calculator with its automatic

arithmetic and its various special functions. This has been a very welcome tool in courses on trigonometry, logarithms and calculus, but it should never have been introduced into school mathematics until trigonometry in Year 9. The main purpose of Years K–8 is learning arithmetic, and the use of calculators there has seriously undermined that purpose. Most secondary mathematics teachers in NSW are complaining about this, despite being told to keep quiet, and despite all the 'studies have shown' of the enthusiasts.

My request is that we fix the situation in NSW with scientific calculators in Years K–8, and that we not make the same mistake with graphics calculators in Years 9–12.

Complexity and simplicity

One cannot but be impressed by the enthusiasm for graphics calculators here. But at the end of the day, we are entrusted with teaching mathematics, not the use of machines. The final objects of our enthusiasm must be things like the understanding of how asymptotes arise, the proof that an angle in a semi-circle is a right angle, the tangent as the limit of a secant, the geometry of tangents to $y = e^x$. These are the things we see as beautiful, and whose beauty we need to convey.

I have heard a lot here about machines allowing us to 'go further', and I think that is a serious misunderstanding. The correct place for machines is complexity; that is the common opinion of mathematicians — you do not reach for the machine for simple objects, except perhaps as an occasional teaching aid.

But the objects and the methods that we teach in our courses do not involve complexity at all — they are simple things. Simplicity is at the core of mathematics itself, and it is the constant discovery of that simplicity which is the source of the beauty we see in our subject. That is why I am a mathematics teacher, and that is the fundamental impulse behind all my day-to-day work.

Reference

Forster, P. & Mueller, U. (1999). Use of graphics calculators in the public examination of calculus: reality and rigour. *Australian Senior Mathematics Journal, 13*(2), 4–20.

Editors' note

The question referred to by Forster and Mueller is reproduced here.

Question 3. Let $f(x) = \dfrac{1}{1 + e^{-x}}$ for $-\infty < x < \infty$.

(a) Determine the range of f.
(b) Show analytically that f is increasing.
(c) Sketch the graph of the inverse of f, f^{-1}, clearly indicating all intercepts and asymptotes.

(Calculus Tertiary Entrance Examination (TEE), WA, 1998).

Graphics Calculators — Issues of Equity

Thelma Perso

Education Department of Western Australia

Most of the issues relating to the use and implementation of graphics calculators in schools are about equity. In Western Australia the various issues have not changed but the degree of emphasis placed on then has varied over time, from the time when the idea of implementation was first mooted to the present.

Prior to and immediately following the introduction of policy allowing the use of graphic calculators in external examinations, the key issue was one of equity of access. Questions such as, 'How can all students afford these computational tools?' were uppermost in the minds of teachers. This issue went all the way to the Minister and was asked in Parliament. As a result, a substantial amount of money was given to government schools on a per capita basis (number of students sitting Tertiary Entrance Examinations in 1998, the first year in which the use of graphics calculators in these examinations was permitted under the policy). The funding was distributed along with the requirement that schools should 'use the money to ensure that all students sitting TEE examinations in Mathematics courses, Physics and Chemistry, should have access to a graphics calculator'.

Schools interpreted this requirement in many different ways, indeed in some schools where graphics calculators had been used for many years and the majority of students owned a graphics calculator, monies were spent purchasing peripheral devices for these. In other schools money was spent in purchasing class sets of calculators which were placed in school libraries and made available for overnight borrowing.

Over time — about one year — this kind of equity issue seemed to disappear as other issues surfaced and became more prominent. Most of these issues fall under the heading of 'Equity of Teacher expertise'.

To markers of the 1999 TEE Mathematics examination scripts, it became obvious that many students had been disadvantaged by the apparent lack of expertise with the technology by their teachers — that is, teachers being unfamiliar with the capabilities of students' calculators and, as a consequence, failing to maximise student efficacy with their graphics calculator.

This lack of teacher expertise can be the result of one or more of the following:

- lack of access to professional development. This is particularly a problem for teachers in country areas since the Mathematical Association of Western Australia (MAWA) is essentially the only ongoing provider of professional development on graphics calculator capabilities, and most of its activities occur in the city.

- the motivation or desire for teachers with access to quality professional develop-

ment who do not wish to attend. There is a need for these teachers to be made aware of the fact that their students may be being disadvantaged by their lack of expertise with the technology. This, I believe, is not a new issue and needs to be brought in line with performance management of teachers by their employer. It raises the question, 'Can exposure to deficiency in an area be used to motivate improved performance?' and may in fact come down to the assumptions made about teacher professionalism.

- the increasing number of teachers of mathematics with little or no training in the teaching of mathematics, let alone with the use of learning technologies for the learning and teaching of mathematics.

The issue of students being disadvantaged by the lack of teacher expertise is not a new one in the mathematics learning area. It is linked with the on-going issue of the need for many primary teachers of mathematics to learn about the effective use of four-function calculators in their classrooms. It is linked to the issue of the need for change in the mindsets of teachers, communities and students in relation to what authentic teaching and learning in mathematics is about. It is about students being disadvantaged by the nature of the pedagogical approaches of the teacher at the front of the room.

In short, the issue of equity of teacher expertise stems from the broader issue of the quality of teachers of mathematics in Australia. The issue needs to be addressed by systems and the Federal Government. Current projects such as the Quality Teacher Program may go a long way to address the issue or they may not. The quality of teachers of mathematics is also effected by the increasing problem teacher shortage in the mathematics learning area. Quality is eroded through using teachers without training or with minimal training in both mathematics and effective mathematics pedagogy.

Problem Solving and Investigative Approaches: *Opportunities and Issues*

Kaye Stacey

University of Melbourne

In this short presentation, I wish to make two points and to illustrate them with two examples. New technology is providing us with wonderful new opportunities to improve the teaching and learning of mathematics. There are opportunities at every year level and through every facility on the new calculators: the home screen, graphing, statistics, programming and on newer models also symbolic manipulation for algebra. On the other hand, the major issue for the profession is to prevent mathematics becoming too empirical, to guard against a tendency towards too much data collection (whether from real situations or from number patterns) and insufficient analysis and reasoning.

The first example is for early secondary school, to show the benefits of having a large home screen.

Students can look for patterns when they follow simple procedures such as the following:

> *Think of a number, add 2, multiply by 5 and subtract 4 times the number you first thought of.*

Using a graphics calculator enables entry errors to show and be corrected and provides a record of recent results, ideal for making and testing conjectures. To make a one-line calculation, students have to convert the step-by-step instructions to standard mathematical notation [(number $+2) \times 5 - 4 \times$ number]. This is a very important skill affecting performance in algebra and arithmetic throughout secondary school, which many students currently do not master — more attention is needed. Students will quickly make the conjecture that the finishing number is ten more than the starting number and this can be used to introduce or consolidate a number of important algebraic ideas, such as the distributive law or collecting like terms. This type of task should not be left at the pattern spotting level.

The second example is from senior secondary mathematics and a worksheet based on this problem can be seen opposite (Dowsey, & Tynan, 1996).

Data has been collected using a temperature probe in a classroom which gives the temperature (T) of a saucepan of water at intervals of 30 seconds for 5 minutes (t is time elapsed). The problem is to predict the temperature later on, say after 30 minutes. Using the built in regression facilities, students can quickly calculate the following regression curves. The value of r-squared shown on the calculator is given after the equation.

24. The Cooling Law Problem(MM&SM3/4)   (KP:9.1-5)

A temperature probe is placed in a saucepan filled with hot water, and the temperature is recorded at 30 second intervals for 5 minutes.   The ambient room temperature is 18.5°. Plot the data and try to fit a rule that will successfully model the temperature of the water after 30 minutes, and justify your choice.

Time (t)	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Temperature (T)	98	91.5	87.5	84	80.5	78	75.5	73	71	69.5	68

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1. Enter the data above into the graphics calculator, and try to fit a linear rule to the data. How well does the line fit the data?

2. Look at the differenced between successive temperature readings in the table above. Why is a linear rule not likely to fit the data well?

3. Try to fit a quadratic rule of the form $T = a(t - h)^2 + k$ to the data by eye. Use the point (5,68) as the vertex of the parabola. Experiment with different functions until you have one which fits the data well.

4. What does your quadratic model predict as the temperature after 30 minutes? Comment on the usefulness of the quadratic model in light of this finding.

5. Another possible model is an exponential one. If such a relationship exists, the ratio between successive terms should be approximately constant. Calculate these ratios and place them in the table below. Does this support the use of an exponential model?

Time (t)	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Temperature (T)	98	91.5	87.5	84	80.5	78	75.5	73	71	69.5	68
Temp(t+0.5)/Temp(t)											

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6. Apply the standard exponential regression model ($y = a*b^x$). What temperature does this predict after 30 minutes? What is worrying about this prediction?

This raises a worrying aspect of fitting regression models to data. An exponential relationship seems plausible, and yet the suggested one has limited predictive power. Indeed other regression models fit the data well, but are not helpful as predictors for the situation (see below).

Linear	*Quadratic*	*Cubic*	*Exponential ($y=a*b^x$)*
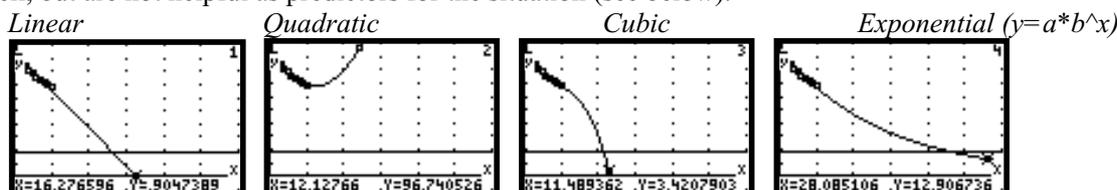
Now try to fit a model by creating a new list of 'excess' temperatures (T – 18.5), and doing an exponential regression on (T – 18.5) versus t.

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7. Enter the 'excess' temperature values into L3, and then reconfigure the scatter plot so that you are plotting L3 (excess temperature) against L1 (time).

8. Now reapply the exponential regression model using the excess temperatures. What temperature does this predict after 30 minutes? How does this compare with the earlier models?

Note: This task involves collecting real data, and attempting to fit a functional model to this data. When the model is to be used for prediction purposes, however, an understanding of the situation is essential in determining the most appropriate model. In the above task, any regression fit will be good for the data given, but only one of the models shown allows sensible extrapolation.

Linear: $T = 93.98 - 5.72t$ (R-squared = 0.961)

Quadratic: $T = 0.80t^2 - 9.72t + 96.98$ (R-squared = 0.997)

Cubic: $T = 97.55 - 11.45t + 1.76\,t^2 - 0.13\,t^3$ (R-squared = 0.999)

Exponential: $T = 94.52\,(0.93)^t$ (R-squared = 0.978)

Plotting the regression lines over the data and also examining the R-squared values shows that the polynomials are an excellent fit, noticeably better than the exponential. Polynomials can fit data in a very convincing way. However, zooming out on the graphs to predict the temperature after 30 minutes shows that they are all useless. The quadratic predicts that the water boils, the other three graphs predict that it freezes. Good modelling proceeds on the basis of selecting a function for a reason, not just because it fits the data. In this case, the quantity that decays exponentially is the difference between the temperature of the water (T) and the ambient temperature (A). A model with predictive power can be found by finding the exponential regression of t against (T-A). This problem exploits the opportunities provided by new technology well, to give real meaning to mathematical ideas. However, it also demonstrates that even practical modelling such as this should not be data-driven, but reason-driven.

Reference

Dowsey, J. & Tynan, D. (1996) *VCE Graphics Calculator Project (Victorian Board of Studies) Workshop Sample Tasks*. Melbourne: University of Melbourne.

Graphics Calculators for Applications and Modelling

Neville Windsor

Hellyer College, Tasmania

Consider the following 'hospitals problem':

> *A town has two hospitals. On the average, there are 45 babies delivered each day in the larger hospital. The smaller hospital has about 15 births each day. Fifty percent of all babies born in the town are boys. In one year each hospital recorded those days in which the number of boys born was 60% or more of the total deliveries for that day in that hospital.*
>
> *Do you think that it's more likely that the larger hospital recorded more such days, that the smaller hospital did, or that the two recorded roughly the same number of such days? (*Kahneman & Tversky, 1972)

What is the point?
What do we want our students to get out of this?

Any teacher who is presenting an applications or modelling scenario to a class is bound to consider this sort of question. In the case of the problem quoted, we would want our students to explore the effect of sample size in a chance phenomenon, and come to an intuitive understanding that 'the larger the sample, the more reliable the results' or 'the larger the sample, the less deviation we expect from the mean'. The advantage we have today, when compared with classes in the past, is that we have the possibility of using graphics calculator (or computer) technology to explore problems such as this instead of relying on theoretical approaches alone. In this way, our students have the opportunity to develop their own intuitions, and thus make sense of the theories. This author has described elsewhere how this problem can be investigated using a spreadsheet (Windsor, 1997) or a graphics calculator (Windsor, 1998).

Watson (2000) discusses the results obtained when a group of graduate students enrolled in a Bachelor of Teaching course were asked to solve the 'hospitals problem'. She found that many mathematics majors lamented their lack of practical experience in statistics. Watson concludes by stressing the importance of both intuitive and mathematical experiences, for both are required to ensure that the mathematics makes sense:

> *... there is no substitute for exposure to practical problems when it comes to understanding statistics* (a PhD student in mathematics, quoted by Watson, 2000).

Moritz and Watson (2000) report on the administration of a survey to 1256 students from grades 6 to 11 concerning their understanding of the probabilities associated with coin tossing. They report that students are both poor at calculating probabilities associated with compound events (tossing several coins) and also poor at estimating these probabilities using intuition. They conclude that students should be given more opportunity to measure probabilities in investigative tasks so that they are less likely to make use of faulty heuristics:

> *Further encouragement to measure chance numerically should reduce reliance on the outcome approach and on intuitions such as the gambler's fallacy...*

Sutton (2000) briefly described the use of graphics calculators in a simple linear modelling context, and reported that he saw more 'lights come on' for his students in a short time than he would normally expect to see in a greater time. He concluded that

> *the real difference is the opportunity students have to reflect on the meaning of their work. They spend little time punching in a few numbers and a lot of time thinking about what it meant.*

Boggs (2000) quotes Bob Hayden in the distinction between practice and experience. Practice is the repetition of some task to increase accuracy and speed, and the final goal is to make it automatic, so that the task can be carried out with little or no thought. The attainment of experience, however, requires variety. Boggs concludes that with the availablity of graphics calculators

> *students no longer need to concentrate so much on practice, which frees them to concentrate on experience — looking at the concepts in a variety of ways, some of them subtle.*

Brown (2000), responding to Boggs, argues that the graphics calculator

> *now allows us to extend the range of tasks that students can perform. The use of modelling and investigational work is ideally suited here and the [students'] skills in mathematics may be less focussed on the calculation of a correlation coefficient but instead on whether or not the calculation makes sense.*

Do not be in too much of a hurry to leap into the technology

A word of warning is required. When graphics calculators (or computers) are introduced into the classroom, our students are very likely to accept them with enthusiasm, but they may not learn what we want them to learn (although they will probably have fun!). Graphics calculators need to be used in ways that assist and support us in our pedagogical role, not take over the role. Technology needs to be used for a purpose, not just for its own sake.

When an investigative task requires simulation, it is a mistake to use the technology too soon, unless students already have considerable experience in such simulation. If, for example, we wish to investigate tossing coins, it is important to begin with real coins. Then, when students have some feel for what is happening, introduce the technology as a way of carrying out a much larger number of coin tosses more quickly and more easily (and more

quietly!). If we move straight to the technology, we run the risk of students not making the connections with the real objects, and not really believing their results. Sometimes students feel that the technology is set up in such a way that it generates the results the teacher wants and thus is not to be taken seriously.

Of course, in any modelling situation, whether or not any technology is used, discussion of the weaknesses in the model, and the assumptions underlying the model, are very important. Such discussion can be very fruitful and can provide the teacher with valuable information about a student's understanding of what has taken place.

Conclusion

The graphics calculator is a powerful tool that can be used in many ways to support the role of the teacher in the classroom. Carefully selected investigative and modelling tasks can be used to help students to develop intuitive understanding of the mathematical concepts involved. Such an intuitive understanding will then support student understanding of any theoretical approach that may be made. Perhaps we will then observe that for many more students, mathematics makes sense.

Note

When the 'hospitals problem' was presented at the conference, those in the audience were asked to indicate, by a show of hands, which of the three alternatives in the final paragraph was correct. Roughly one-third of those present indicated that they thought the smaller hospital would have more days with 60% or more boys, another one-third felt the two hospitals would have about the same number of days, a small number felt the larger hospital would have more such days, and a considerable number failed to 'vote'! It is interesting to observe that even in an audience made up mainly of highly experienced teachers of mathematics, the majority was unable to nominate the correct answer — the smaller hospital.

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Conference Participants

Participants

The organisers sincerely thank all participants for their contributions.

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